Principles of Mathematics 12

Geometric Series

$$\sum_{k=4}^{7}(3k+2)^2$$

Lesson 1

Geometric Sequences and the General Term

Principles of Math 12 EXPLAINED!

By Barry Mabillard

Geometric Series

Part I – a & r

GEOMETRIC SEQUENCES:

2, 4, 8, 16, 32...

A = FIRST TERM
R = COMMON RATIO

- THE COMMON RATIO IS A NUMBER (OR VARIABLE) THAT YOU MULTIPLY A TERM BY IN ORDER TO GET THE NEXT TERM.
- THE COMMON RATIO MUST BE THE SAME FOR ALL TERMS, OTHERWISE THE SEQUENCE IS NOT GEOMETRIC.
- THE COMMON RATIO CAN BE FOUND BY DIVIDING A TERM BY THE PREVIOUS TERM.

Example 1: Which of the following sequences are geometric? State the a & r values for those which are.

a) *3, 6, 9, 12...* *Not geometric (Adding 3, not multiplying)*

b) *2, 4, 8, 16, ... a = 2, r = 2*

c) $12, 3, \frac{3}{4}, \ldots \quad a = 12, r = \frac{1}{4}$

d) *x, 2x, 3x ...* *Not geometric (Adding x, not multiplying)*

e) $\frac{5}{3}, \frac{10}{9}, \frac{20}{27} \ldots \quad a = \frac{5}{3}, r = \frac{2}{3}$

The common ratio is what you <u>multiply</u> a term by in order to arrive at the next term.

For example, in the sequence 27, 9, 3, the common ratio is 1/3 since multiplication by one third will bring you to the next term.

QUESTIONS:

1) 81, 27, 9…
2) 4, 8, 16 …
3) 14, 7, 7/2, …
4) x-3, 9-3x, 9x-27 …
5) $\frac{2}{5}, \frac{4}{5}, \frac{8}{5} \ldots$
6) $3, 3\sqrt{3}, 9 \ldots$
7) $\frac{1}{x}, x, x^3 \ldots$

ANSWERS:

1) $a = 81, r = \frac{1}{3}$
2) a = 4, r = 2
3) a = 14, r = ½
4) a = x-3, r = -3
5) $a = \frac{2}{5}, r = 2$
6) $a = 3, r = \sqrt{3}$
7) $a = \frac{1}{x}, r = x^2$

Geometric Series

Part II – General Term

GENERAL TERM: THE GENERAL TERM IS AN ALGEBRAIC EXPRESSION THAT ALLOWS YOU TO FIND THE VALUE OF ANY TERM.

a = 2

2	4	8	16	32	....	$2(2)^{n-1}$
n=1	n=2	n=3	n=4	n=5		n=n
$t_1 = 2$	$t_2 = 4$	$t_3 = 8$	$t_4 = 16$	$t_5 = 32$		$t_n = t_n$

N REPRESENTS THE TERM POSITION
T_N REPRESENTS THE VALUE OF THE N^{TH} TERM

The formula used to find any term in the sequence is: $t_n = ar^{n-1}$

Example 1: Find the common ratio for the following and state the value of t_{12}

a) 5, 10, 20...

a = 5

r = 10 ÷ 5

r = 2

$t_n = ar^{n-1}$

$t_n = 5(2)^{n-1}$

$t_{12} = 5(2)^{12-1}$

$t_{12} = 10240$

b) $\frac{4}{7}, \frac{6}{7}, \frac{9}{7}...$

a = $\frac{4}{7}$

r = $\frac{6}{7} \div \frac{4}{7}$

r = $\frac{6}{7} \bullet \frac{7}{4}$

r = $\frac{3}{2}$

$t_n = ar^{n-1}$

$t_n = \frac{4}{7}\left(\frac{3}{2}\right)^{n-1}$

$t_{12} = \frac{4}{7}\left(\frac{3}{2}\right)^{12-1}$

$t_{12} = 49.43$

QUESTIONS:

1) 5, 10, 20… t_n, t_9

2) 6000, 3000, 1500… t_n, t_6

3) $\frac{4}{7}, \frac{16}{35}, \frac{64}{125}...$ t_n, t_{11}

4) $-1, \frac{2}{3}, -\frac{4}{9}...$ t_n, t_{10}

5) $2x^2, 4x^3, 8x^4$ t_n, t_9

ANSWERS:

1) $t_n = 5(2)^{n-1}$ $t_9 = 1280$

2) $t_n = 6000\left(\frac{1}{2}\right)^{n-1}$ $t_6 = 187.5$

3) $t_n = \frac{4}{7}\left(\frac{4}{5}\right)^{n-1}$ $t_{11} = 0.06136$

4) $t_n = -1\left(-\frac{2}{3}\right)^{n-1}$ $t_{10} = 0.02601$

5) $t_n = 2x^2(2x)^{n-1}$ $t_9 = 512x^{10}$

Geometric Series

Part III – Algebraic Methods

Example 1: If $t_3 = 1$ and $t_8 = \frac{1}{32}$, find the general term.

Write the sequence as follows:

$$\_\_, \_\_, 1, \_\_, \_\_, \_\_, \_\_, \frac{1}{32}$$

Treat 1 like it's the first term

If 1 is considered the first term, then 1/32 is the sixth term.

Use: $a = 1$, $t_6 = 1/32$, $n = 6$ in the formula $t_n = ar^{n-1}$

$t_n = ar^{n-1}$

$\frac{1}{32} = 1(r)^{6-1}$

$1 = 32r^5$

$\frac{1}{32} = r^5$

$r = \sqrt[5]{\frac{1}{32}}$

$r = \frac{1}{2}$

Now use the value of r to finish the sequence

$$4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$$

If $a = 4$ and $r = \frac{1}{2}$

The general term is:

$t_n = 4\left(\frac{1}{2}\right)^{n-1}$

Example 2: Insert two geometric means between 4 and $\frac{32}{27}$

Find an equation using:

$a = 4$, $t_4 = \frac{32}{27}$ and $n = 4$

$t_n = ar^{n-1}$

$\frac{32}{27} = 4r^{4-1}$

$32 = 108r^3$

$\frac{32}{108} = r^3$

$r = \sqrt[3]{\frac{32}{108}}$

$r = \frac{2}{3}$

Now that you know the common ratio, fill in the rest of the sequence

$$4, \frac{8}{3}, \frac{16}{9}, \frac{32}{27}$$

Example 3: If $2a - 2$, and $2a + 2$, and $5a + 1$ form a geometric sequence, determine the value of each of the terms to the nearest hundredth.

If the terms form a geometric sequence, the common ratio will be the same.

Recall that you find a common ratio by dividing a term by the previous term.

As a result,

$\frac{t_2}{t_1} = \frac{t_3}{t_2}$

Cross multiply and then solve the quadratic by graphing and finding the x - intercepts.

$\frac{2a+2}{2a-2} = \frac{5a+1}{2a+2}$

$(2a+2)(2a+2) = (2a-2)(5a+1)$

$4a^2 + 8a + 4 = 10a^2 - 8a - 2$

$0 = 6a^2 - 16a - 6$ Divide both sides by 2

$0 = 3a^2 - 8a - 3$

$0 = (3a+1)(a-3)$

$a = -\frac{1}{3}, 3$

Plug each of the solutions into the sequence and obtain actual numbers.

2a - 2, 2a + 2, and 5a + 1

a = -1/3 gives: $-\frac{8}{3}, \frac{4}{3}, -\frac{2}{3}$

a = 3 gives: 4, 8, 16

Geometric Series

Part III – Algebraic Methods

QUESTIONS:

1) If $t_4 = 54$ and $t_7 = 1458$ find the value of t_9

2) If $t_5 = 4$ and $t_8 = \frac{32}{27}$ find the value of t_9

3) Insert three geometric means between 8 and $\frac{1}{32}$

4) Insert two geometric means between 3 and $\frac{81}{8}$

5) If 2x - 2, 2x + 4, and 7x - 4 form a geometric sequence, determine the value of each of the terms.

ANSWERS:

1) 13122

2) $r = \frac{2}{3}, t_9 = \frac{64}{81}$

3) r = 1/4
The sequence is $8, \mathbf{2}, \mathbf{\frac{1}{2}}, \mathbf{\frac{1}{8}}, \frac{1}{32}$

4) $r = \frac{3}{2}$
The sequence is $3, \mathbf{\frac{9}{2}}, \mathbf{\frac{27}{4}}, \frac{81}{8}$

5) For x = 4:
The terms are 6, 12, 24

For $x = -\frac{1}{5}$:

The terms are $-\frac{12}{5}, \frac{18}{5}, -\frac{27}{5}$