PRINCIPLES OF MATHEMATICS 12

Transformations Practice Exam

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Transformations Practice Exam

Use this sheet to record your answers

1. 11. 20.
2. NR 3. 21.
3. 13. 22.
NR 2. 14. 23.
4. 15. 24.
5. 16. 25.
7. NR 4. 27.
8. NR 5. 28.
9. 18. 29.
10. 19.
Transformations Practice Exam

1. If $f(x) = x^2 - 1$, then a function with the same domain and range as $f(x)$ is

A. $g(x) = f(x - 1)$
B. $g(x) = f(x) - 1$
C. $g(x) = f^{-1}(x)$
D. $g(x) = \frac{1}{f(x)}$

Use the following information to answer the next five questions.

The graph of a function is shown below.

The graph shows a parabola with a vertex at the origin and opening upwards. The x-intercepts are at $x = -2$ and $x = 2$.

Numerical Response

1. If the transformation $y = f(-2x)$ is applied, the value of the largest x-intercept is, to the nearest whole number, ________.
2. If the graph of $f(x)$ is transformed to a new function $y - 4 = f(x - 2)$, then the range of the new graph is

A. $y \geq -8$
B. $y \geq -6$
C. $y \geq -4$
D. $y \geq 4$

3. The number of invariant points in the graph of $\frac{1}{f(x)}$ is

A. 2
B. 4
C. 6
D. Impossible to determine

**Numerical Response**

2. If the graph of $y = f(x)$ is stretched vertically about the line $y = -5$ by a factor of 3, then the new $y$-intercept is $(0,b)$. The value of $b$ is _______.

4. A true statement regarding the graph of $y = f^{-1}(x)$ is

A. An $x$-intercept occurs at the point $(0, -10)$
B. The graph is not a function
C. The point $(0, -2)$ becomes the point $(2, 0)$
D. The graph has no $x$-intercepts
5. The graph of \( f(x) \) and the graph of \( g(x) = f^{-1}(x) \) are correctly represented by which of the following pairs of graphs?

A.  
B.  
C.  
D.  

Use the following information to answer the next question.
Use the following information to answer the next question.

The graph of \( y = f(x) \) is shown below.

6. The graph of \( f\left(\frac{1}{2}x\right) \) is correctly represented by which of the following?

A.  

B.  

C.  

D.
7. The graph of \( y = -2f(x+5) \) is the same as the graph of

A. The graph of \( y = f(x) \) reflected about the x-axis, then shifted five units right, then stretched vertically by a factor of 2 about the x-axis.

B. The graph of \( y = f(x) \) reflected about the y-axis, then stretched vertically by a factor of \( \frac{1}{2} \) about the x-axis, then shifted five units left.

C. The graph of \( y = f(x) \) stretched by a factor of 2 about the y-axis, reflected about the y-axis, then shifted five units left.

D. The graph of \( y = f(x) \) stretched by a factor of 2 about the x-axis, reflected about the x-axis, then shifted five units left.

Use the following information to answer the next question.

The graph of \( f(x) = \sqrt{x} \) is shown below

8. The statement which best describes the graph of \( g(x) = f(-x) \) is

A. \( g(x) \) is defined for all values of \( x \)

B. \( g(x) \) is defined for \( x \geq 0 \)

C. \( g(x) \) has a range of \( y \geq 0 \)

D. \( g(x) \) is undefined for all values of \( x \)
9. The point (8, -5) is on the graph of \( y = f(x) \). If the transformation \( y = f(2x + 4) \) is applied, then the new point is

A. (2, -5)  
B. (20, -5)  
C. (0, -5)  
D. (4, -1)

Use the following information to answer the next question.

The graph of \( f(x) \) is shown below

![Graph of f(x)]

10. A true statement regarding the graph of \( y = \frac{1}{f(x)} \) is

A. The reciprocal graph has a vertical asymptote  
B. The reciprocal graph is not a function  
C. There are two invariant points  
D. There are two x-intercepts in the reciprocal graph
11. The graph of \( f(x) = x^2 - 2 \) undergoes the transformation \( f(x+1) \).
If a student wishes to graph the transformed function in their calculator, the equation that gives the correct graph is

A. \( x^2 - 1 \)
B. \( x^2 - 3 \)
C. \( (x+1)^2 - 2 \)
D. \( (x-1)^2 - 2 \)

12. If the graph of \( f(x) = x^2 \) is transformed to the graph of \( y + 2 = f(x+1) \), then a true statement regarding the two graphs is

A. The domain, but not the range, is the same.
B. The range, but not the domain, is the same.
C. Both the domain and range are the same
D. The domain and range are both different

Use the following information to answer the next question.

The graph of \( m(x) \) is shown, along with three possible reflections.

A student knows the following reflections were used:
1. \( y = -f(x) \)
2. \( y = f(-x) \)
3. \( y = -f(-x) \)

**Numerical Response**

3. The reflections used to produce the graphs in quadrants II, III, & IV, respectively, are _____, _____, and _____.

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Use the following information to answer the next two questions.

The graphs of $f(x)$, $g(x)$, and $h(x)$ are shown below.

13. The transformation applied to $f(x)$ in order to obtain $g(x)$ is

A. A reflection across the $x$-axis, then a vertical stretch by a factor of $\frac{1}{2}$ about the $y$-axis.
B. A reflection across the $y$-axis, then a vertical stretch by a factor of $\frac{1}{2}$ about the $x$-axis.
C. A vertical stretch by a factor of 2 about the line $y = 2$, then a reflection across the $y$-axis.
D. A vertical stretch by a factor of $\frac{1}{2}$ about the line $y = 2$, then a reflection across the $y$-axis.

14. The transformation applied to $f(x)$ in order to obtain $h(x)$ is

A. $h(x) = -f(x-1) - 8$
B. $h(x) = f(x-1) - 8$
C. $h(x) = f(x+1) + 8$
D. $h(x) = f(x+1) - 8$
15. The graph of \( y = f(x) \) is horizontally stretched by a factor of 3 about the \( y \)-axis, reflected in the \( x \)-axis, then translated four units right and two units up. The transformed graph is represented by

A. \( y = -f\left(\frac{1}{3}(x-4)\right) + 2 \)
B. \( y = -f\left(3(x-4)\right) + 2 \)
C. \( y = f\left(-3(x-4)\right) + 2 \)
D. \( y = f\left(\frac{1}{3}(-x-4)\right) + 2 \)

Use the following information to answer the next question.

The graph of a function is shown below

16. If the reflection \( y = f(-x) \) is applied to the graph, the invariant point is

A. I
B. II
C. III
D. IV
Use the following information to answer the next question.

The graph of \( f(x) \) is shown below. The domain is \(-2 \leq x \leq 2\).

17. The graph of \( x = f(y) \) is represented by which of the following graphs?

A. 

B. 

C. 

D.
4. The function $f(x) = 2x^3 - 4x^2 + 3x - 5$ is multiplied by a constant $b$ to apply a vertical stretch to the graph. If the transformed graph passes through the point (-2, -129), then the value of $b$ is ________.

Use the following information to answer the next question.

Three points that lie on a function $f(x)$ are shown below.

![Graph of a function with points A(-2, 2), B(1, -1), and C(5, 4).]

5. If the function is transformed by $y - 4 = 2f(x)$, then the new $y$-values of points A, B, and C are, respectively, ______, ______, and ______.

18. A transformation is applied to the graph of $y = f(x)$ such that the point (2, 2) is invariant. A transformation that can produce this result is

A. $y = 2f(x)$
B. $y = -f(x)$
C. $y = \frac{1}{f(x)}$
D. $y = f^{-1}(x)$
19. A vertical asymptote in the graph of \( \frac{1}{f(x)} \) is located at

A. \( x = -5 \)
B. \( x = -2 \)
C. \( y = -2 \)
D. \( x = 8 \)

20. If \( y \) is replaced with \( \frac{1}{3} y \) in the equation \( y = f(x) \), then the resulting transformation on the graph will be

A. A vertical stretch by a factor of \( \frac{1}{3} \) about the \( x \)-axis
B. A vertical stretch by a factor of 3 about the \( x \)-axis
C. A horizontal stretch by a factor of \( \frac{1}{3} \) about the \( y \)-axis
D. A horizontal stretch by a factor of 3 about the \( y \)-axis
A function \( f(x) \) is transformed to produce the graph of \( g(x) = f(x - 7) + 8 \). If the graph is further transformed by moving it two units left and one unit down, then the new graph can be written as \( h(x) = f(x - a) + b \). The numerical values of \( a \) and \( b \) are, respectively, ______, and ______.

Use the following information to answer the next question.

Points on the graph of \( y = f(x) \) are shown below

21. If the graph is stretched vertically about the line \( y = -3 \) by a factor of 2, then the new coordinate of point C is \((4, m)\). The value of \( m \) is

A. 0  
B. 1  
C. 3  
D. 6
Use the following information to answer the next three questions.

Points on the graph of \( y = f(x) \) are shown below

![Graph of \( y = f(x) \)](image)

22. The number of vertical asymptotes found in the graph of \( y = \frac{1}{f(x)} \) is

A. 0  
B. 1  
C. 3  
D. 4

23. The number of invariant points found in the graph of \( y = \frac{1}{f(x)} \) is

A. 0  
B. 1  
C. 3  
D. 4

24. If the graph is transformed to \( g(x) = f(2x - 4) \), then point A becomes \((m, 1)\). The value of \( m \) is

A. 0  
B. 1  
C. 3  
D. 4
25. The domain of \( f(x) \) is \( x \leq 3 \). If the transformation \( g(x) = f(x + 10) - 2 \) is applied, then the new domain of the function is

A. \( x \leq -10 \)  
B. \( x \leq -7 \)  
C. \( x \geq -10 \)  
D. \( x \geq -7 \)

26. A point on the graph of \( f(x) \) is (-3, 4). If the transformation \( y = f(3x - 6) - 1 \) is applied, then the new coordinates of the point are

A. (1, 3)  
B. (-1, 4)  
C. (-15, 3)  
D. (5, 3)

27. The function \( f(x) = x^2 - 5x + 6 \) is multiplied by a constant \( b \) to apply a vertical stretch to the graph. If the transformed graph passes through the point (8, 15), then the value of \( b \) is \( \boxed{\text{_____}} \).

A. 4  
B. \( \frac{1}{4} \)  
C. 2  
D. \( \frac{1}{2} \)

28. The graph of \( y = (x + 1)^2 \) undergoes the transformation \( y = f^{-1}(x) \).

A true statement regarding the transformed graph is

A. The transformed graph is the reciprocal of the original  
B. The transformed graph is not a function  
C. The transformed graph has the same domain and range as the original graph  
D. The vertex of the parabola is invariant
29. The graph of \( f(x) \) is horizontally stretched about the line \( x = 2 \) by a factor of \( \frac{1}{2} \). The vertex on the transformed graph is located at the point

A. \((-4, 0)\)
B. \((0, 0)\)
C. \((1, 0)\)
D. \((0, -1)\)
A triangular arch tops a doorway in an ancient building, as shown below.

Written Response – 10%

1. Draw a graph that represents the figure shown above, with the centre of the doorway at the origin.
- Determine the equation of the triangular arch, and write it in the form 
  \[ y = b|x - p| + q \], where \( b \) is the vertical stretch factor, and \((p, q)\) is the vertex. 
  Also, state the domain and range of the triangular arch.

- If the height of the arch is increased from 14 m to 16 m (\textit{while keeping the base of the triangular arch at the same level}) describe what happens to each of the parameters \( b, p, \) and \( q \).
Use the following information to answer the next question.

The graph of \( y = f(x) \) is shown below.

Written Response – 10%

2. In the space provided below, draw in the graph of \( y = 2f(x) \) and write a description of the transformation.
• In the space provided below, draw in the graph of \( y = -f(x) \) and write a description of the transformation.

\[ f(x) \]

• In the space provided below, draw in the graph of \( y = \frac{1}{f(x)} \) and write a description of the transformation.

\[ f(x) \]
The graph of \( y = f(x) \) is shown below.

**Written Response – 10%**

3. List the invariant points in the graph of \( y = -f(x) \)

3. List the invariant points in the graph of \( y = f(-x) \)

3. List the invariant points in the graph of \( y = f(2x) \)

3. List the invariant points in the graph of \( y = \frac{1}{f(x)} \)

3. List the invariant points if the graph is stretched vertically about the line \( y = \frac{1}{2} \) by a factor of 2

3. List the invariant points in the graph of \( y = -f(-x) \)