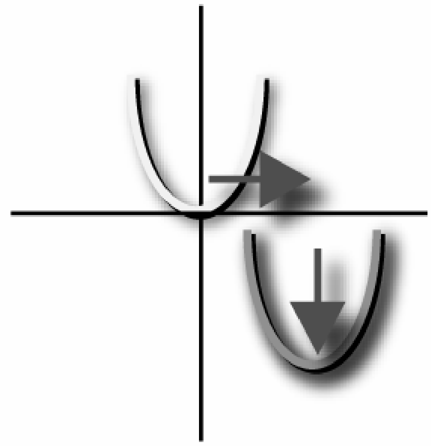
Pre - Calculus Math 40S:

TRANSFORMATIONS



LESSON 4

Other Transformations

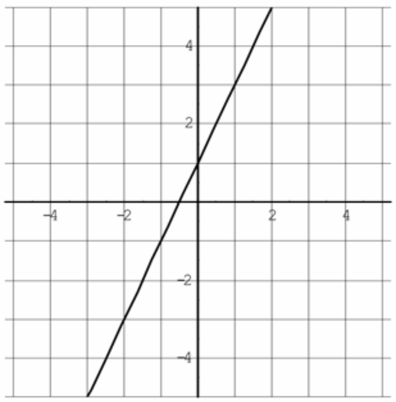
Pre - Calculus
Math 40S
EXPLAINED!

By
Barry
Mabillard

Transformations Lesson 4
Part 1: Absolute Value Graphs

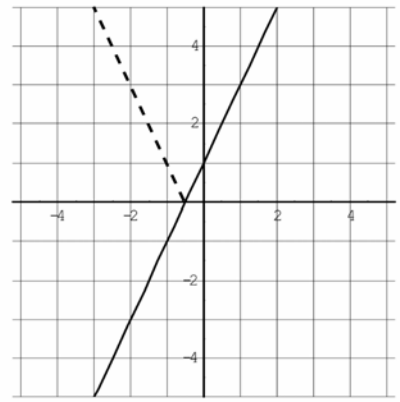
Absolute Value Graphs: **The absolute value symbol in y = |f(x)| means all y-coordinates must be made positive.**

Example 1: Draw the graph of y = |f(x)|



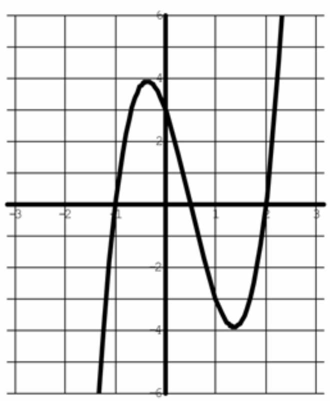
The region of the graph below the x-axis is reflected above.

The region of the graph already above the x-axis is unaffected.



Questions: **For each of the following, draw in y = |f(x)|:**

1)



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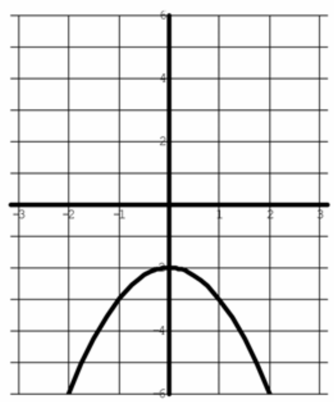

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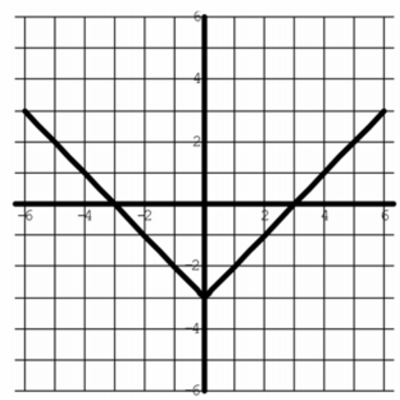
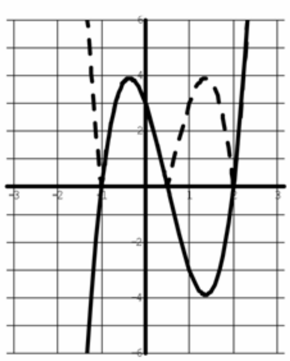


Answers:

1)



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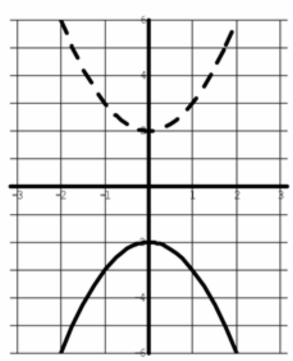
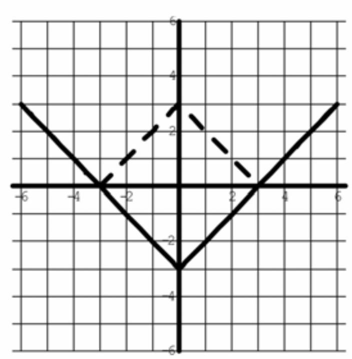
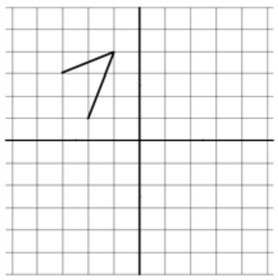


3)



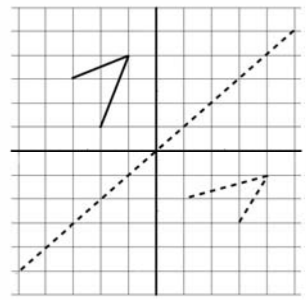
Inverses: An inverse is defined as $x = f(y)$ or $y = f^{-1}(x)$, and may be obtained by interchanging the x & y values. As such, inverses are reflected over the line $y = x$.

Example 1: Draw in the inverse of the given graph.



First draw in the line y=x. Now graph the inverse by interchanging the x & y values.

e.g. The point (-2, 1) will become (1, -2)



Example 2: Determine the equation of the inverse to $f(x) = 3x - 4$

First rewrite f(x) as y : $y = 3x - 4$

Now interchange x & y : $x = 3y - 4$

Solve for the new y : $x + 4 = 3y$

$$y = \frac{x+4}{3}$$

Inverses are written with the notation $f^{-1}(x)$, so express the final answer as : $f^{-1}(x) = \frac{x+4}{3}$

Example 3: Find the equation of the inverse to $f(x) = (x - 3)^2 - 4$

$y = (x - 3)^2 - 4$

$x = (y - 3)^2 - 4$

$x + 4 = (y - 3)^2$

$\pm\sqrt{x+4} = y - 3$ Remember that you need the ± when taking a square root

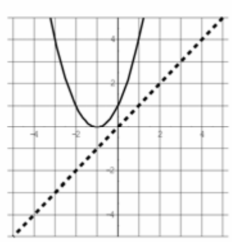
$f^{-1}(x) = \pm\sqrt{x+4} + 3$

For inverses, the invariant points are on y = x

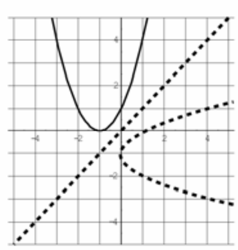
Example 4: How would you restrict the domain of $y = (x + 1)^2$ such that the inverse is a function?

First find the equation of the inverse : $f^{-1}(x) = \pm\sqrt{x} - 1$

Graph the original



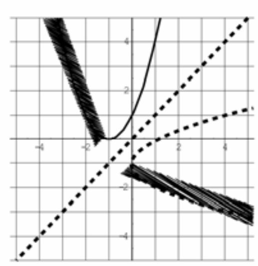
Now draw the inverse



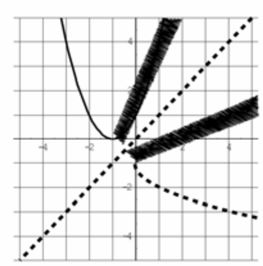
Notice that the inverse is NOT a function, since it doesn't pass the vertical line test.

To make the inverse a function, we need to restrict the domain of the original so the inverse *will* pass the vertical line test.

The domain of the original could be restricted to $x \geq -1$ This will force the inverse to be a function.



Or, the domain of the original could be restricted to $x \leq -1$ This will also force the inverse to be a function.

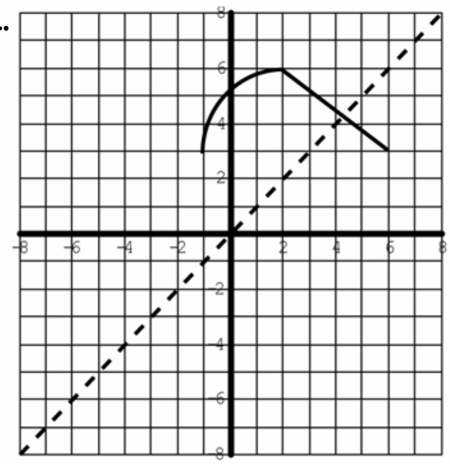


Transformations Lesson 4
Part II: Inverses

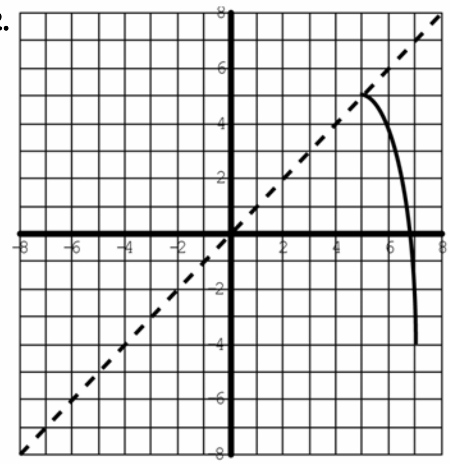
Questions:

Draw in the inverse for each of the following graphs.

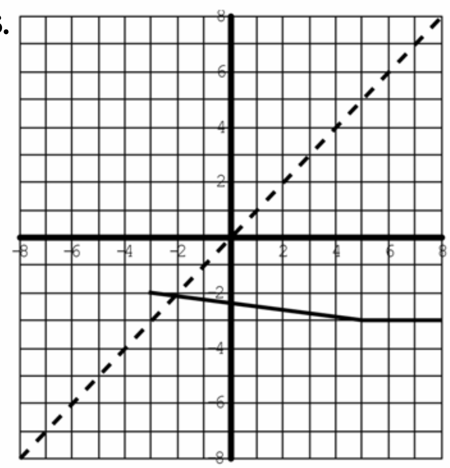
1.



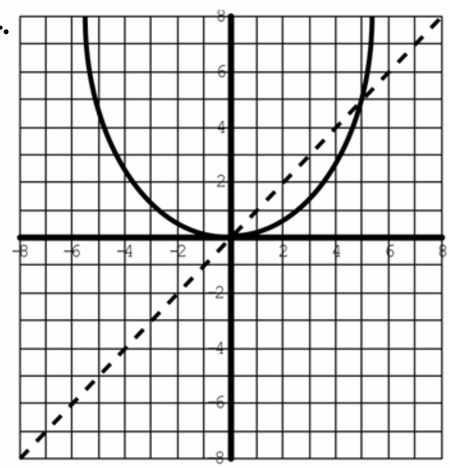
2.



3.



4.



5) Find the equation of the inverse for each of the following:

a) $f(x) = 4x - 5$

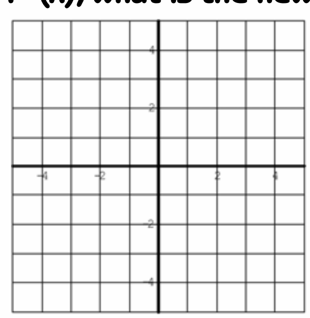
b) $f(x) = x^2 - 4$

c) $f(x) = (x + 2)^2$

d) $f(x) = \sqrt{x - 1}$

6) If the point (-3, 4) undergoes the transformation $y = f^{-1}(x)$, what is the new point?

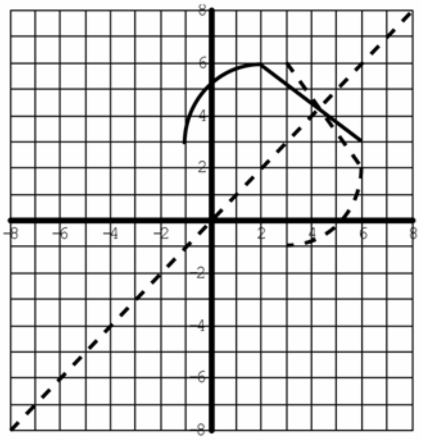
7) How would you restrict the domain of $y = (x - 2)^2$ such that the inverse $f^{-1}(x) = \pm\sqrt{x} + 2$ is a function?



Transformations Lesson 4
Part II: Inverses

Answers:

1.



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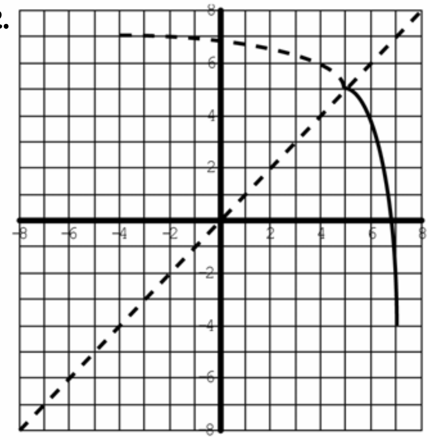

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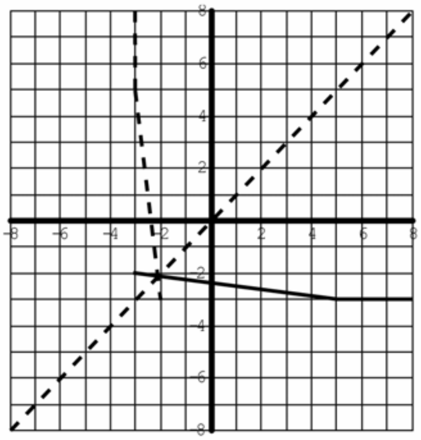

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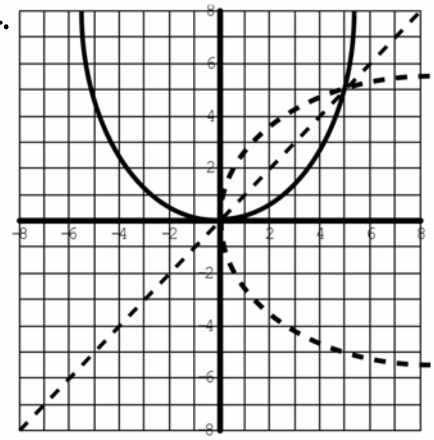


5.

a) $f^{-1}(x) = \dfrac{x+5}{4}$

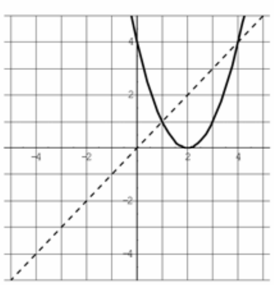
b) $f^{-1}(x) = \pm\sqrt{x+4}$

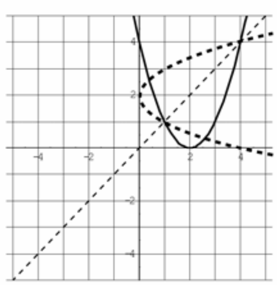
c) $f^{-1}(x) = \pm\sqrt{x} - 2$

d) $f^{-1}(x) = x^2 + 1$

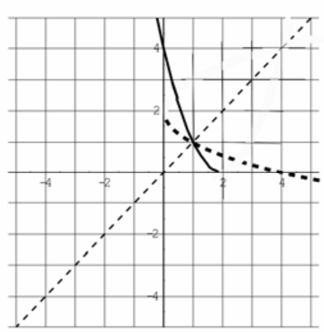
6. The new point is found by interchanging x & y, (4, -3)

7. First graph the function & the inverse.

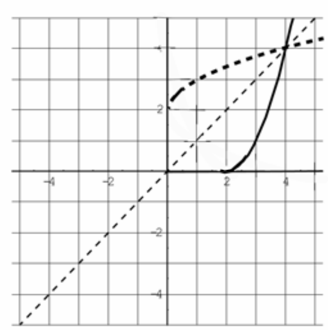




If the domain of the original is restricted to $x \le 2$, this will force the inverse to be a function.



Or, the domain of the original could be restricted to $x \ge 2$. This will also force the inverse to be a function.



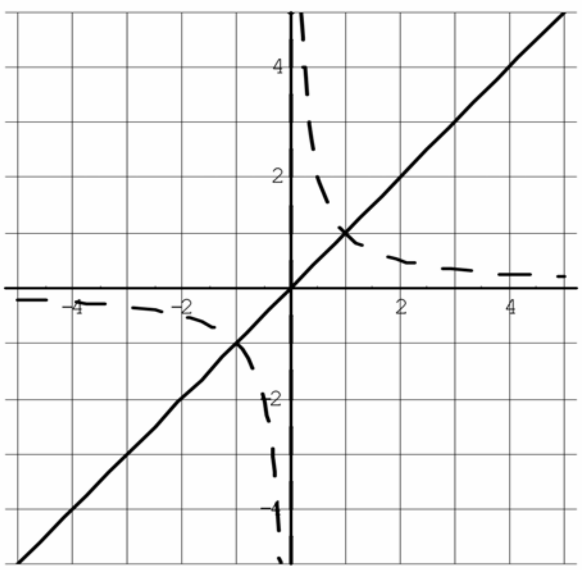
Reciprocal Functions: A reciprocal function is represented by $y = \frac{1}{f(x)}$

> Note that $\frac{1}{f(x)}$ and $f^{-1}(x)$ do NOT mean the same thing! The first is a reciprocal graph, the second is an inverse graph. They are very different.

Example 1: Draw $y = x$ and the reciprocal graph, $y = \frac{1}{x}$

Black: $y = x$

Dashed: $y = \frac{1}{x}$



Obtain the values in this column by using the y-values of the original graph. | Reciprocal the numbers from the first column.

$f(x)$	$\frac{1}{f(x)}$
-4	-0.25
-2	-0.5
-1	-1
-0.5	-2
-0.1	-10
0	UNDEFINED
0.1	10
0.5	2
1	1
2	0.5
4	0.25

Remember what happens in basic division of numbers:

As the denominator becomes larger, the resulting number becomes smaller.

$\frac{1}{2} = 0.5$ $\frac{1}{4} = 0.25$ $\frac{1}{100} = 0.01$

As the denominator becomes smaller, the resulting number becomes larger.

$\frac{1}{0.5} = 2$ $\frac{1}{0.25} = 4$ $\frac{1}{0.01} = 100$

Remember that dividing by zero is undefined, so at the x-intercepts of the original, the reciprocal graph has no corresponding point.

Indicate this on the graph by drawing a ***vertical asymptote***, since the graph will approach this line from both sides but never actually reach it.

Note the following characteristics of reciprocal graphs:

1) The reciprocal graph will always be on the same side of the x-axis as the original.
If the original is above the x-axis, the reciprocal is also above.
If the original is below, the reciprocal is also below.

2) The vertical asymptotes are drawn at the x-intercepts of the original.
The equation of a vertical line is of the form: x = constant.
In the graph above, the equation of the vertical asymptote is x = 0

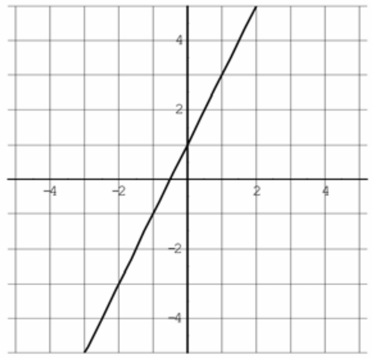
3) Horizontal asymptotes are may also by present in the reciprocal graph.
The equation of a horizontal line is of the form: y = constant.
In the graph above, the equation of the horizontal asymptote is y = 0

4) Invariant points are points that don't change position when a transformation is applied.
In reciprocal graphs, the invariant points are located at y = -1 and y = 1.

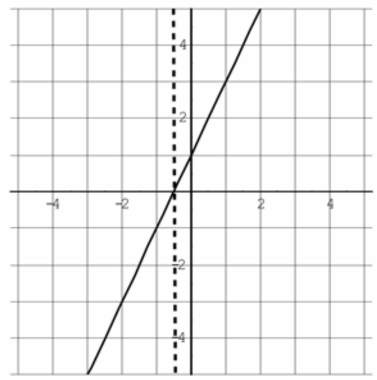
TRANSFORMATIONS LESSON 4
PART III: LINEAR RECIPROCALS

<u>Example 2:</u> Draw the graph of $y = 2x + 1$ and it's reciprocal, $y = \frac{1}{2x+1}$

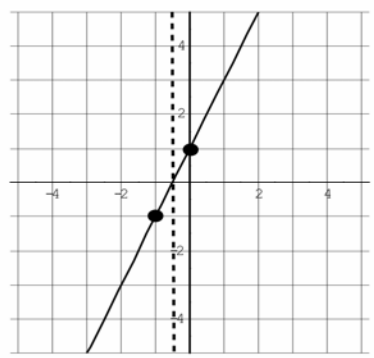
<u>Step 1:</u> Draw $y = 2x + 1$



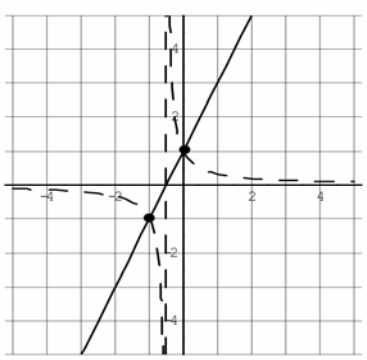
<u>Step 2:</u> Draw in the vertical asymptote at the x-intercept.



<u>Step 3:</u> Place dots at the invariant points (wherever y = ±1 on the original)



<u>Step 4:</u> Draw in the reciprocal graph.

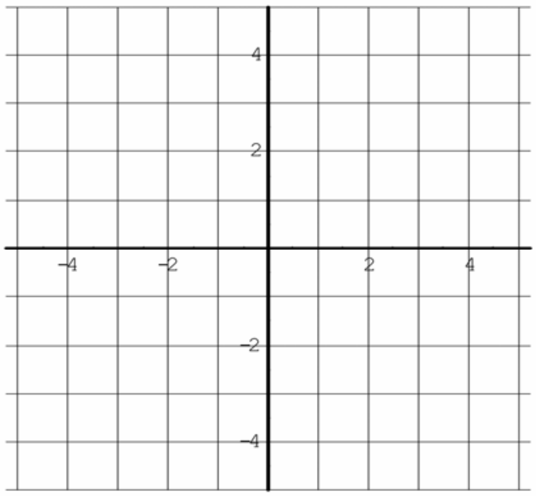


Domain: $x \neq -0.5$
Range: $y \neq 0$

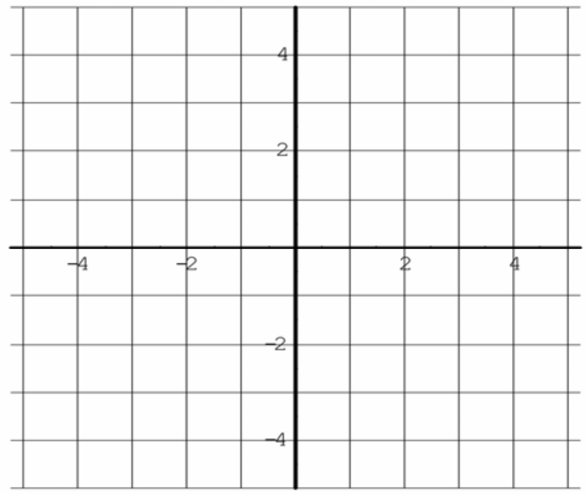
Part III: Linear Reciprocals

Questions: **Draw the original & reciprocal graphs for each of the following functions. State the domain and range for the reciprocal.**

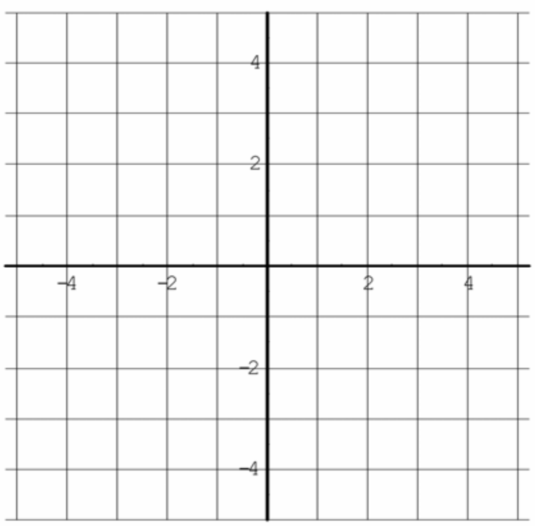
1) $y = -x$



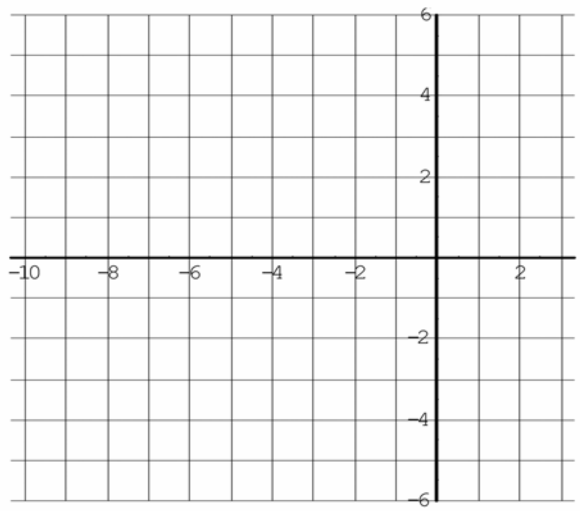
2) $y = x - 3$



3) $y = 2x + 2$

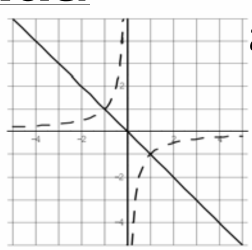


4) $y = -\frac{1}{2}x - 2$



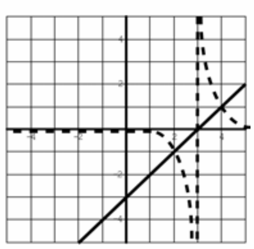
Answers:

1)



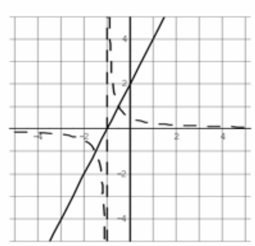
Domain: $x \neq 0$
Range: $y \neq 0$

2)



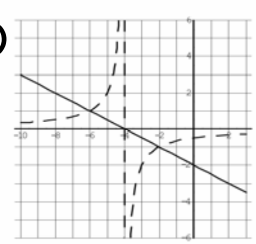
Domain: $x \neq 3$
Range: $y \neq 0$

3)



Domain: $x \neq -1$
Range: $y \neq 0$

4)

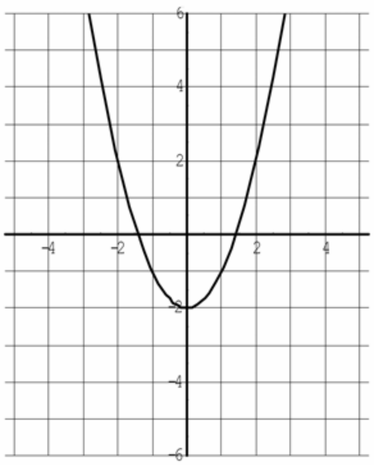


Domain: $x \neq -4$
Range: $y \neq 0$

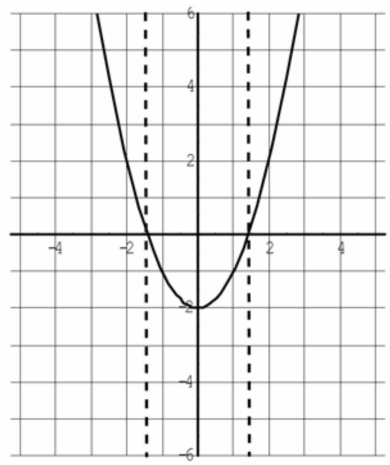
Reciprocals of Non-Linear Graphs: In these graphs, you have to be careful about where a reciprocal passes through, touches, or completely misses the original graph.

<u>Example 1:</u> Draw the reciprocal of $y = x^2 - 2$

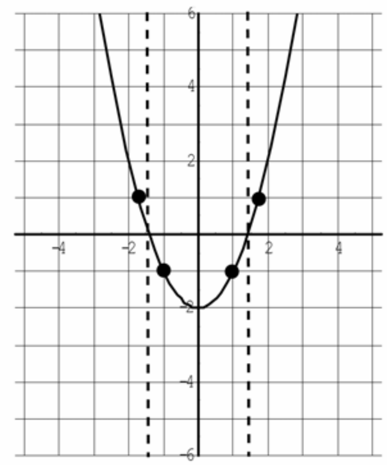
<u>Step 1:</u> Draw $y = x^2 - 2$



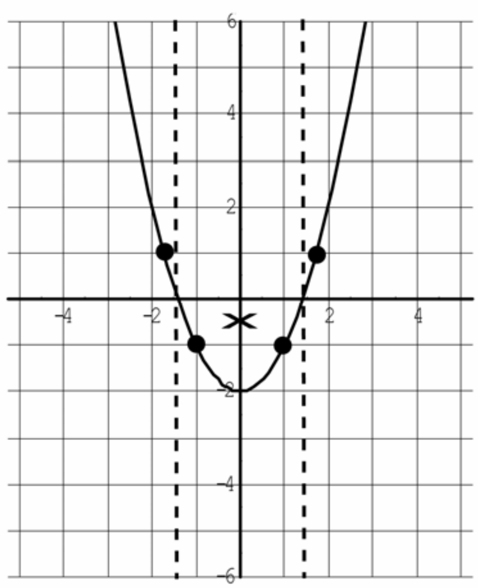
<u>Step 2:</u> Draw in vertical asymptotes at the x-intercepts



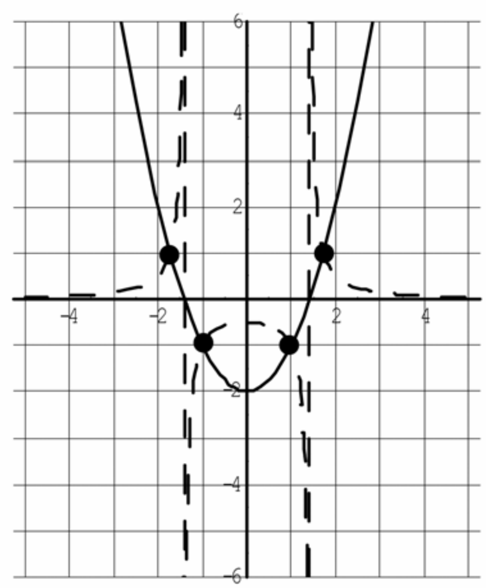
<u>Step 3</u>: Draw in dots at the invariant points.



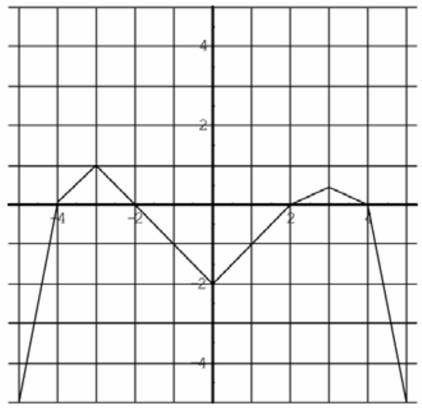
<u>Step 4:</u> Look at the bottom tip of the parabola. The y-value there is -2, so the reciprocal value is -0.5. Put a tick there since the reciprocal graph must pass through that point.



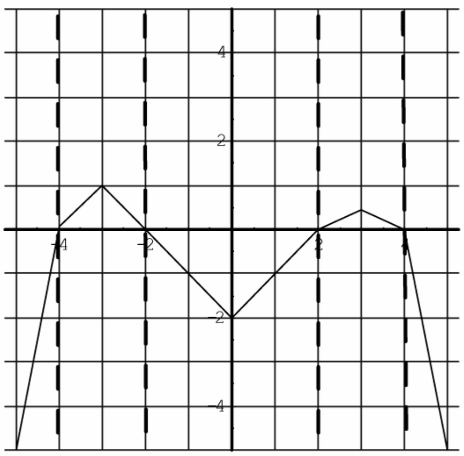
<u>Step 5:</u> Draw in the graph



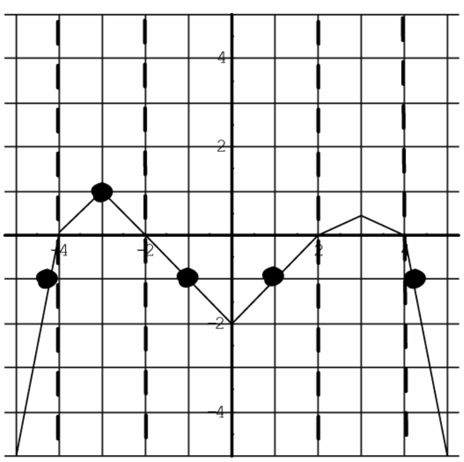
Example 2: **Draw in the reciprocal of the following graph:**



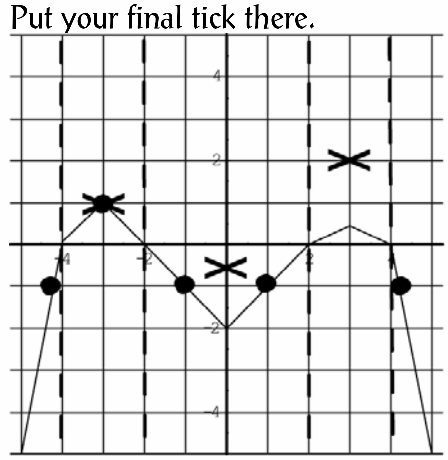
Step 1: Draw in the asymptotes



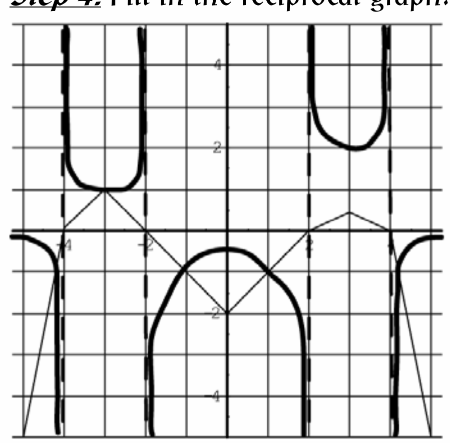
Step 2: Draw in dots at all invariant points.



Step 3:
The first tip is at y = 1, and the reciprocal of this is also 1. Place a tick there.
The next tip is at -2, with a reciprocal of -0.5. Place another tick there.
The last tip is at 0.5, and the reciprocal is 2. Put your final tick there.

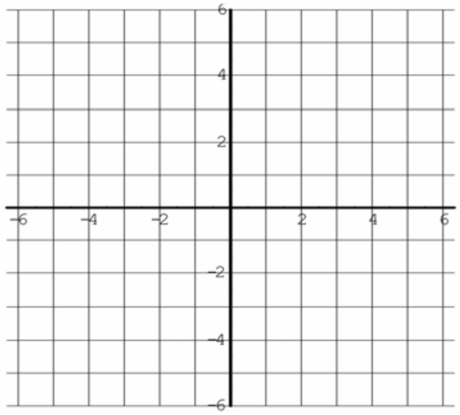


Step 4: Fill in the reciprocal graph.

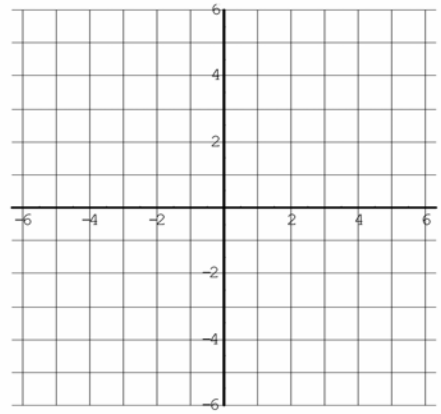


Questions: **Draw the original & reciprocal graph for each of the following:**

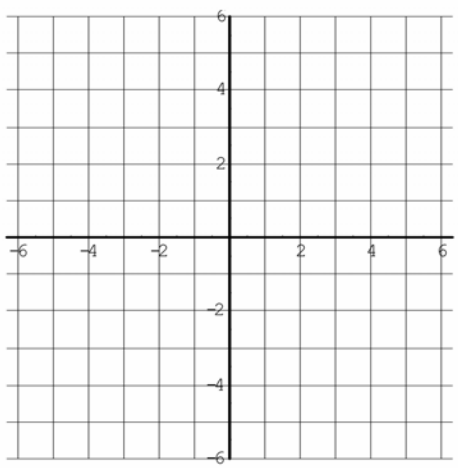
1. $y = \frac{1}{2}x^2 - \frac{1}{2}$



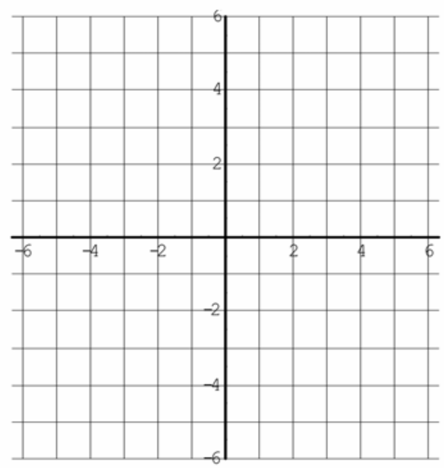
2. $y = -(x-2)^2 + \frac{1}{2}$



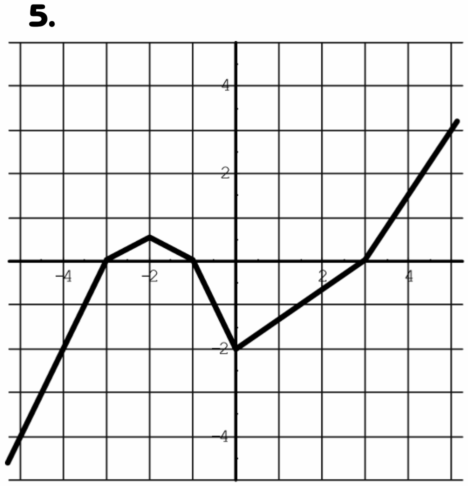
3. $y = -(x+2)^2 + 1$



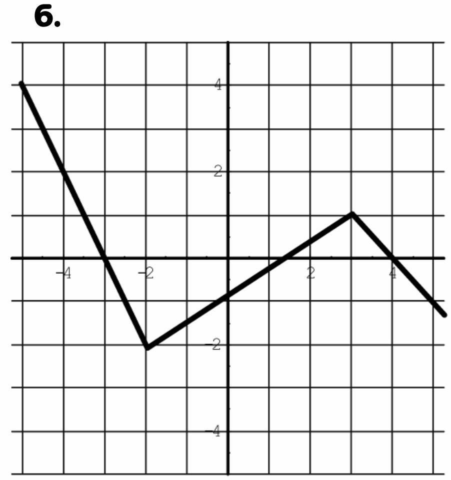
4. $y = (x-3)^2 - 2$



5.

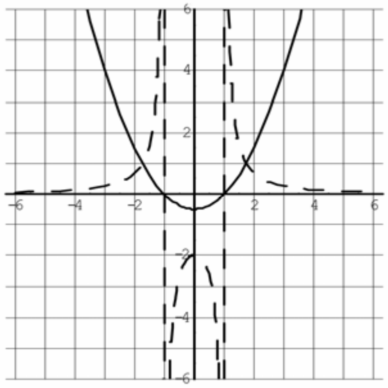


6.

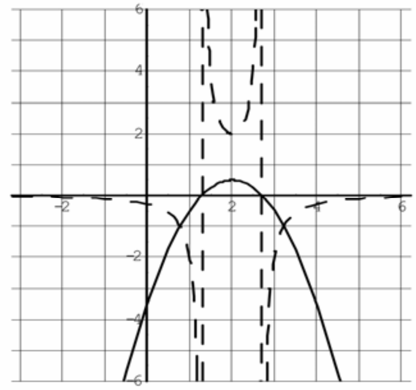


Answers:

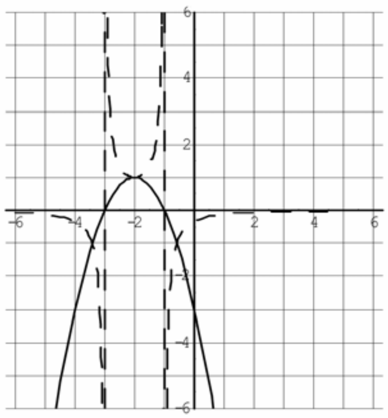
1.



2.



3.



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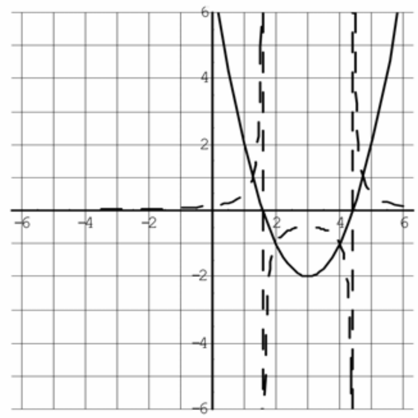
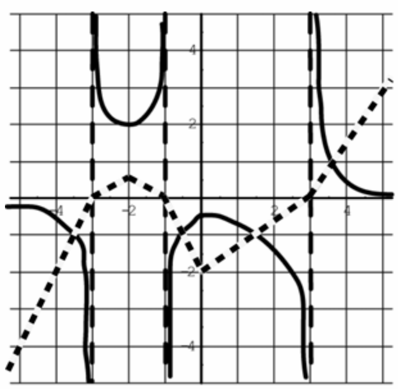


5.



6.