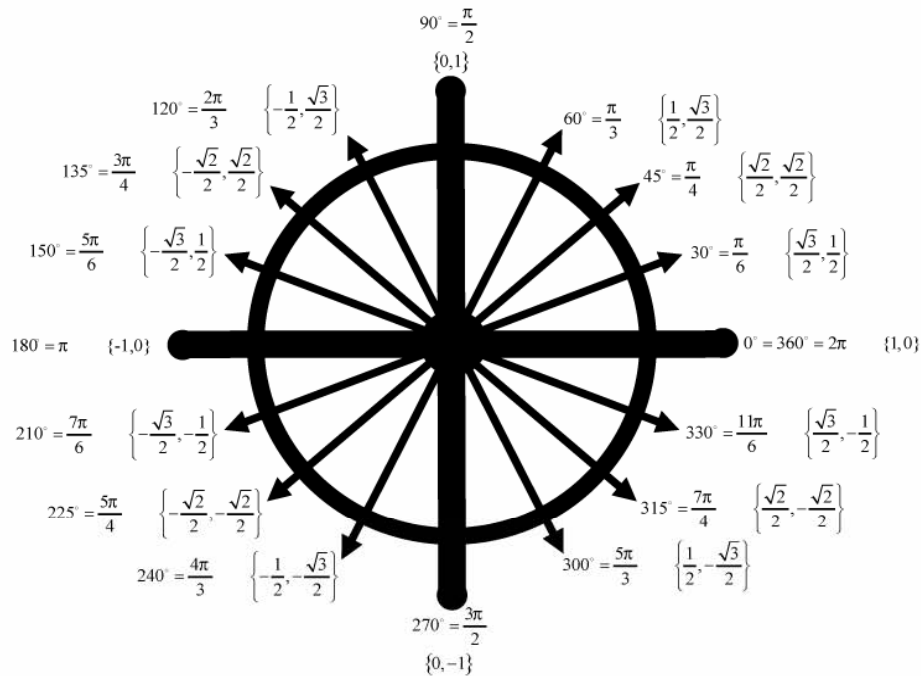


Pure Math 30:

TRIGONOMETRY I



LESSON ONE

Types of Angles

Pure Math
30:

EXPLAINED!

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TRIGONOMETRY LESSON ONE

PART I TYPES OF ANGLES

Memorize the following definitions:

Terminal arm: The line representing where the angle is located in a plane. The terminal arm always comes out of the origin.

Standard Angle: The angle between the positive x-axis and the terminal arm, rotating counterclockwise.

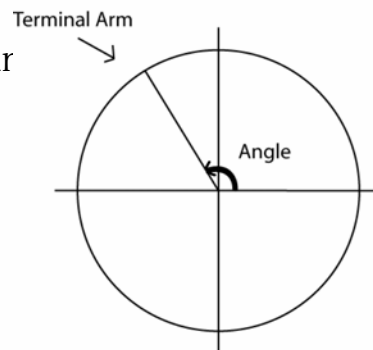
Principal angle: An angle between 0° and 360° .

Co-terminal angle: An angle less than 0° or greater than 360° that shares the same position as a principal angle.

Negative angle: Angles obtained from rotating the terminal arm clockwise.

Reference angle: The angle between the terminal arm and the x-axis.

General Solution: A formula that gives all possible co-terminal angles.



Example 1: Find both a positive & negative co-terminal angle for 72° .

The diagram illustrates the process of finding co-terminal angles for 72° . It starts with a circle on the left showing a 72° angle in the first quadrant. Two jagged arrows point to two other circles. The top circle shows a 432° angle, which is one full revolution plus 72° . The bottom circle shows a -288° angle, which is one full clockwise revolution minus 72° .

To find the positive co-terminal angle, add 360° to make one complete counterclockwise revolution.
 $72^\circ + 360^\circ = 432^\circ$

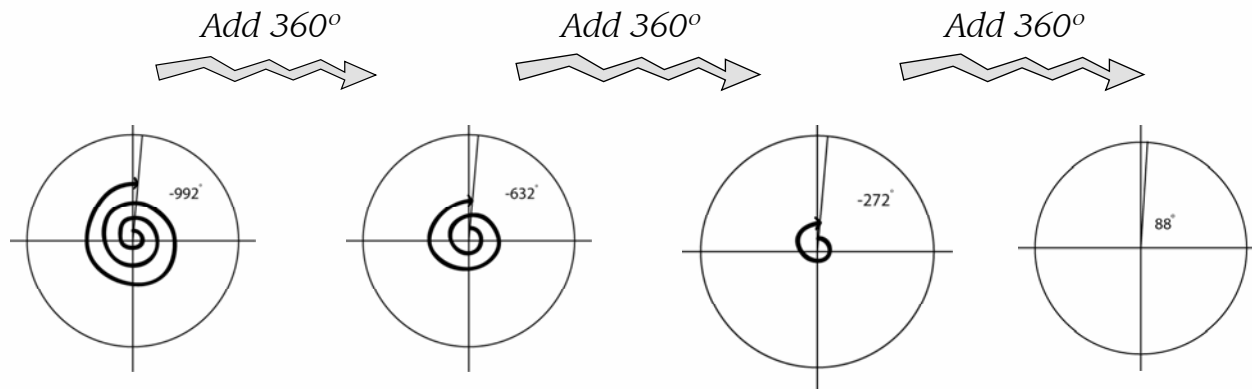
To find the negative co-terminal angle, subtract 360° to make one complete clockwise revolution.
 $72^\circ - 360^\circ = -288^\circ$

TRIGONOMETRY LESSON ONE

PART I TYPES OF ANGLES

Example 2: Given the co-terminal angle -992° , find the principal angle.

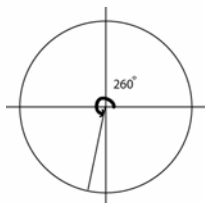
We need to “unwind” our way back to between 0° and 360° by making revolutions of 360° .



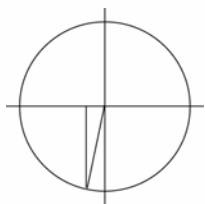
The principal angle is 88°

Example 3: Find the reference angle for 620° .

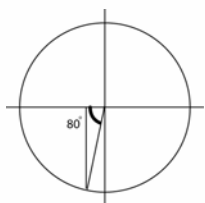
First find the principal angle. $620^\circ - 360^\circ = 260^\circ$



Connect a line to the x-axis.



Now determine the angle inside the triangle.
($260^\circ - 180^\circ = 80^\circ$)



The reference angle is 80°

Example 4: Find the general solution for all co-terminal angles of 101° , and state the value of the 14th positive co-terminal angle.

The general solution must include all possible revolutions of 360° , so we use the formula:

$$\text{Principal Angle} \pm n(360^\circ)$$

For this question, it's $101^\circ \pm n(360^\circ)$

When $n = 1$, we can either add 360° to get a positive co-terminal, or subtract 360° for a negative co-terminal.

When $n = 2$, we can either add 720° to get a positive co-terminal, or subtract 720° for a negative co-terminal.

The 14th positive co-terminal angle occurs when $n = 14$.

$$\begin{aligned} & 101^\circ + 14(360^\circ) \\ & \text{*choose the plus since we want the positive angle.} \\ & = 101^\circ + 5040^\circ \\ & = 5141^\circ \end{aligned}$$

TRIGONOMETRY LESSON ONE

PART I TYPES OF ANGLES

1) For each of the following principal angles, state a negative & positive co-terminal angle:

- a) 47°
- b) 102°
- c) 321°
- d) 225°

2) For each of the following co-terminal angles, find the principal angle:

- a) -1023°
- b) 541°
- c) 888°
- d) -361°

3) Find the reference angle for each of the following:

- a) 992°
- b) -502°
- c) -1337°
- d) 600°

4) Find the principal angle for each of the following reference angles:

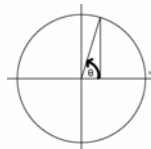
- a) 43° , Quadrant III
- b) 72° , Quadrant IV
- c) 89° , Quadrant II
- d) 1° , Quadrant III

5) State the general solution for all co-terminal angles, and find the 10th positive angle for the following:

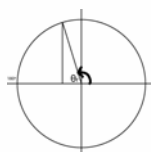
- a) 52°
- b) 131°
- c) 300°

Reference Angles Quickchart:

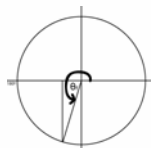
Q1: Reference = Principal



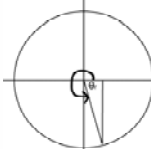
Q2: Reference = $180^\circ - \text{Principal}$



Q3: Reference = Principal - 180°



Q4: Reference = $360^\circ - \text{Principal}$



Answers

- | | | |
|-------------------------------|----------------|-----------------|
| 1. | 3. | 5. |
| a) -313° & 407° | a) 88° | a) 3652° |
| b) -258° & 462° | b) 38° | b) 3731° |
| c) -39° & 681° | c) 77° | c) 3900° |
| d) -135° & 585° | d) 60° | |
| 2. | 4. | |
| a) 57° | a) 223° | |
| b) 181° | b) 288° | |
| c) 168° | c) 91° | |
| d) 359° | d) 181° | |

TRIGONOMETRY LESSON ONE

PART II ANGLE CONVERSIONS

There are three different ways of expressing angles:

Degrees: There are 360° in a circle.

Radian Decimals: 1 radian is equal to 57.3° . There are 6.28 radians in a circle.

Radian Fraction: Expressing radian values as a fraction involving π .
There are 2π radians in a circle.

Throughout the unit, you will be working with all three types of angles.

The following examples show how to convert from one type to another.

Example 1: Convert 201° to a radian decimal:

To convert from degrees \rightarrow radian decimals, multiply by the fraction $\frac{\pi}{180^\circ}$

$$201^\circ \times \frac{\pi}{180^\circ} = 3.51 \text{ rad}$$

Example 2: Convert 225° to a radian fraction.

To convert from degrees \rightarrow radian fractions, multiply by the fraction $\frac{\pi}{180^\circ}$

$$225^\circ \times \frac{\pi}{180^\circ} = \frac{225^\circ \times \pi}{180} = \frac{5\pi}{4}$$

Divide these numbers in your TI-83, then Math \rightarrow Frac

Remember These Conversions:

Radians \rightarrow Degrees: Multiply by $\frac{180^\circ}{\pi}$

Degrees \rightarrow Radians: Multiply by $\frac{\pi}{180^\circ}$

Example 3: Convert 3.24 radians to degrees:

To convert from radian decimal \rightarrow degrees, multiply by the fraction $\frac{180^\circ}{\pi}$

$$3.24 \times \frac{180^\circ}{\pi} = 185.6^\circ$$

Example 4: Convert $\frac{\pi}{4}$ radians to degrees:

To convert from radian fractions \rightarrow degrees, multiply by the fraction $\frac{180^\circ}{\pi}$

$$\frac{\pi}{4} \times \frac{180^\circ}{\pi} = \frac{\pi}{4} \times \frac{180^\circ}{\pi} = 45^\circ$$

TRIGONOMETRY LESSON ONE

PART II ANGLE CONVERSIONS

Example 5: Convert $\frac{7\pi}{6}$ radians to a radian decimal.

To convert from radian fraction \rightarrow radian decimal, simply type the fraction into your calculator:

$$7 \times \pi \div 6 = 3.67 \text{ rads}$$

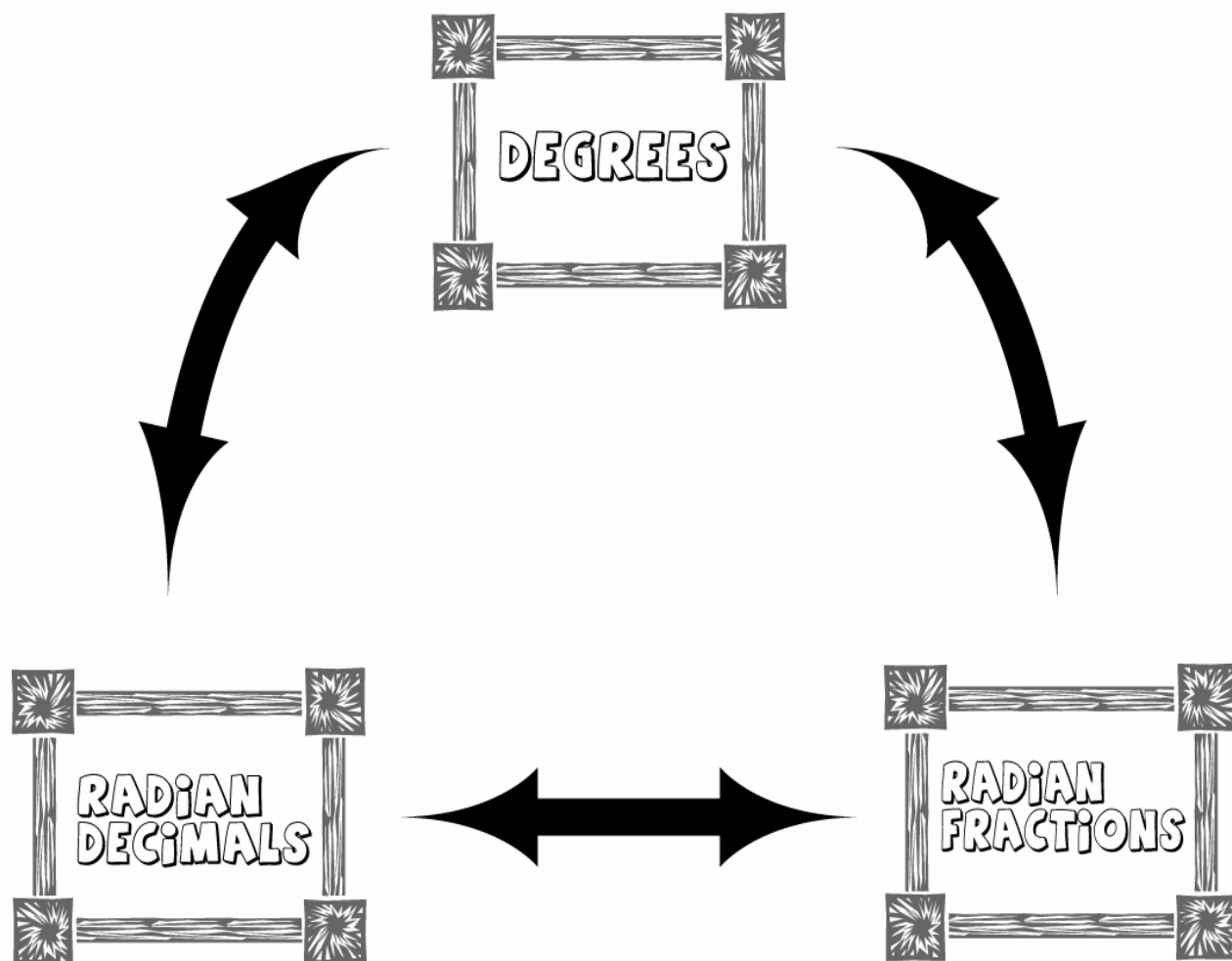
Example 6: Convert 0.7854 radians to a radian fraction.

To convert from radian decimal \rightarrow radian fraction, first convert to degrees, then to a radian fraction.

$$0.7854 \times \frac{180^\circ}{\pi} = 45^\circ$$

$$45^\circ \times \frac{\pi}{180^\circ} = 45^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{4}$$

Divide 45° by 180° and then math \rightarrow frac to get $\frac{1}{4}$.



PURE MATH 30: EXPLAINED!

TRIGONOMETRY LESSON ONE

PART II ANGLE CONVERSIONS

1. Convert the following from degrees \rightarrow radian decimal:

- a) 342°
- b) 62°
- c) 100°

2. Convert the following from degrees \rightarrow radian fraction:

- a) 300°
- b) 60°
- c) 150°

3. Convert the following from radian decimal \rightarrow degrees:

- a) 0.02
- b) 2.51
- c) 1.33

4. Convert the following from radian fraction \rightarrow degrees:

- a) $\frac{2\pi}{3}$
- b) $\frac{11\pi}{6}$
- c) $\frac{3\pi}{2}$

5. Convert the following from radian fraction \rightarrow radian decimal:

- a) $\frac{5\pi}{4}$
- b) $\frac{5\pi}{3}$
- c) $\frac{\pi}{2}$

6. Convert the following from radian decimal \rightarrow radian fractions:

- a) 2.6180
- b) 6.2832
- c) 5.2360

Answers

1.

- a) 5.97
- b) 1.08
- c) 1.75

2.

- a) $\frac{5\pi}{3}$
- b) $\frac{\pi}{3}$
- c) $\frac{5\pi}{6}$

3.

- a) 1.15°
- b) 143.8°
- c) 76.2°

4.

- a) 120°
- b) 330°
- c) 270°

5.

- a) 3.93
- b) 5.24
- c) 1.57

6.

- a) $\frac{5\pi}{6}$
- b) 2π
- c) $\frac{5\pi}{3}$

TRIGONOMETRY LESSON ONE

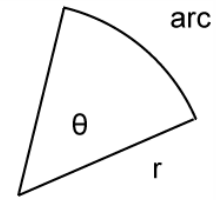
PART III Arc Length

We can find the length of an arc by using the formula $a = r\theta$

a = length of arc

r = radius of circle

θ = angle of arc. (Radians ONLY)



Example 1: What is the length of arc if a radius of 4 m sweeps through 185.6° ?

First, convert the angle to a radian decimal. $185.6^\circ \times \frac{\pi}{180^\circ} = 3.24$

Next, plug your numbers into the formula

$$a = r\theta$$

$$a = (4)(3.24)$$

$$a = 12.96 \text{ m}$$

Example 2: What is the angle in degrees if a radius of 2.3 m cuts an arc of 6.1 m ?

First solve for θ :

$$\theta = \frac{a}{r}$$

$$\theta = \frac{6.1}{2.3}$$

$$\theta = 2.65$$

Now convert this radian decimal to degrees: $2.65 \times \frac{180^\circ}{\pi} = 152^\circ$

1) Find the arc length given the following information:

a) radius = 10 m & $\theta = 5.4$ radians

b) diameter = 16 m & $\theta = 22^\circ$

2) Find the radius given the following:

a) arc length = 16 m & $\theta = 123^\circ$

b) arc length = 23 m & $\theta = \frac{5\pi}{4}$

3) Find the angle (in degrees) given the following:

a) arc length = 2 m & radius = 1 m

b) arc length = 5.2 m & radius = 3 m

Answers

1)

a) 54 m

b) 3.07 m

2)

a) 7.45 m

b) 5.86 m

3)

a) 114.6°

b) 99.3°