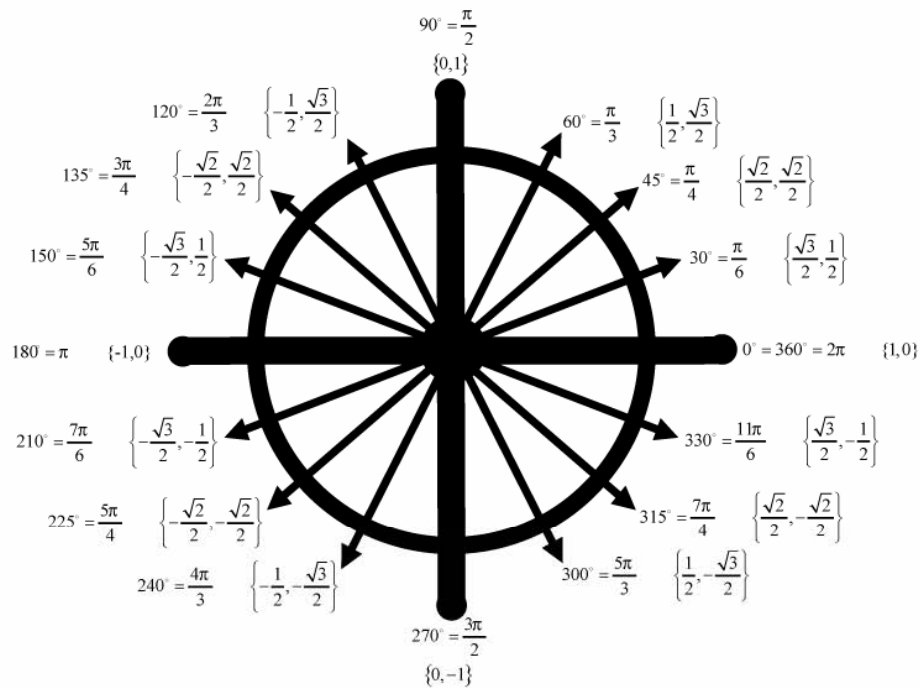


Pure Math 30:

TRIGONOMETRY I



Lesson Nine

Other Trigonometric Graphs

Pure Math
30:

EXPLAINED!

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Mabillard

Trigonometry Lesson 9

Part I - Graphing Other Trig Functions

Graphing $y = \tan \theta$ & $y = \cot \theta$

The vertical lines you see are called *asymptotes*. They are places where the graph is undefined.

Recall that $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\cot \theta = \frac{\cos \theta}{\sin \theta}$

$\tan \theta$ is undefined at the angles where $\cos \theta$ is

equal to zero. $\left[\frac{\pi}{2}, \frac{3\pi}{2} \right]$

Likewise, $\cot \theta$ is undefined whenever $\sin \theta$ is equal to zero. $[0, \pi]$

a-value: We only use the term *amplitude* in describing the graphs of $\sin \theta$ and $\cos \theta$. The other four trig graphs are not "closed in", they go up & down forever. So, we simply call the a-value the *vertical stretch*.

b-value & period: IMPORTANT! The period of a basic $\tan \theta$ or $\cot \theta$ graph is π , not 2π like $\sin \theta$ and $\cos \theta$. Thus, we have the following formulas:

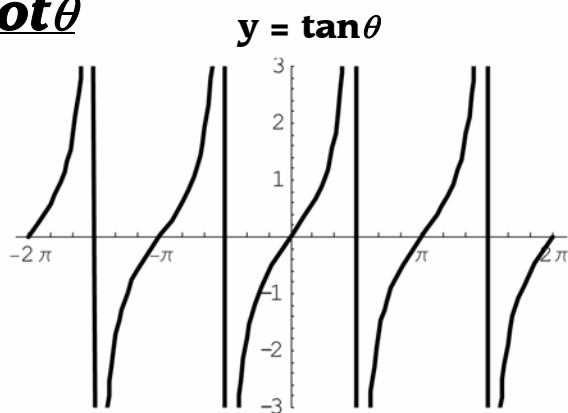
$$\text{Period} = \frac{\pi}{b} \quad \text{and} \quad b = \frac{\pi}{\text{Period}}$$

c-value: No difference from $\sin \theta$ and $\cos \theta$, but remember to move your asymptotes if you shift the graph.

d-value: No difference from $\sin \theta$ and $\cos \theta$.

We always write the general solution of $\tan \theta$ & $\cot \theta$ asymptotes in the following way:

$x = \text{Angle of first positive asymptote} \pm n(\text{Period})$

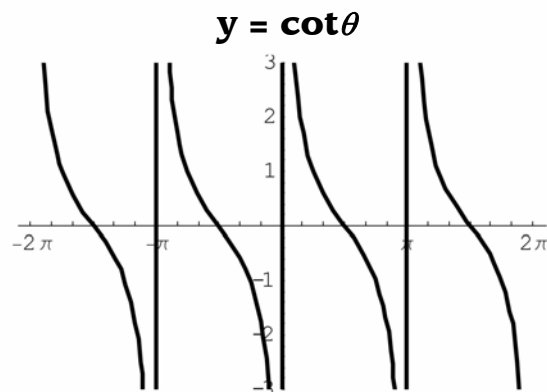


For $\tan \theta$, we can see from the graph that the first positive asymptote occurs at $\frac{\pi}{2}$.

All asymptotes are exactly π units away from each other.

The general equation of the asymptotes is:

$$x = \frac{\pi}{2} \pm n\pi$$



For $\cot \theta$, we can see from the graph that the first positive asymptote occurs at 0, and all asymptotes are exactly π units away from each other.

The general equation of the asymptotes is:

$$x = 0 \pm n\pi, \text{ or simply, } x = \pm n\pi$$

Trigonometry Lesson 9

Part I - Graphing Other Trig Functions

Graphing $y = \csc\theta$ & $y = \sec\theta$

As with the previous graphs, the vertical lines you see are asymptotes, and they occur where the graph is undefined.

Recall that $\csc\theta = \frac{1}{\sin\theta}$ and $\sec\theta = \frac{1}{\cos\theta}$

At the angles where $\cos\theta$ is equal to zero, the graph is undefined

Likewise, $\cot\theta$ is undefined whenever $\sin\theta$ is equal to zero.

a-value: Vertical stretch, makes graphs taller and more narrow.

b-value & period: For the reciprocal graphs $\csc\theta$ and $\sec\theta$, they have a period of 2π , so we can use the formulas we're used to:

$$\text{Period} = \frac{2\pi}{b} \quad \text{and} \quad b = \frac{2\pi}{\text{Period}}$$

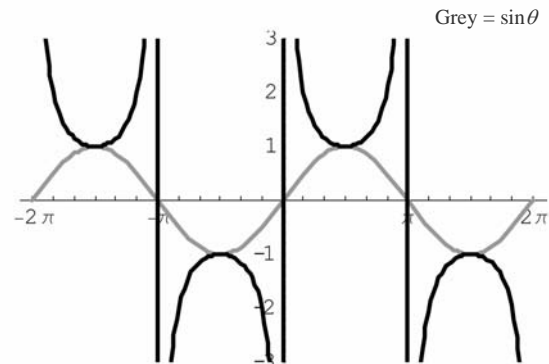
c-value: No difference from $\sin\theta$ and $\cos\theta$, but remember to move your asymptotes if you shift the graph.

d-value: No difference from $\sin\theta$ and $\cos\theta$.

We always write the general solution of $\csc\theta$ & $\sec\theta$ asymptotes in the following way:

$$x = \text{Angle of first positive asymptote} \pm n \left(\frac{\text{Period}}{2} \right)$$

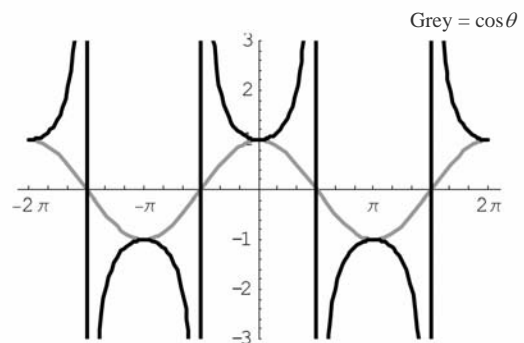
$$y = \csc\theta$$



For $\csc\theta$, we can see from the graph that the first positive asymptote occurs at 0, and all asymptotes are exactly π units away from each other. The general equation of the asymptotes is:

$$x = 0 \pm n\pi, \text{ or simply, } x = n\pi$$

$$y = \sec\theta$$



For $\sec\theta$, we can see from the graph that the first positive asymptote occurs at $\frac{\pi}{2}$. All asymptotes are exactly π units away from each other.

The general equation of the asymptotes is:

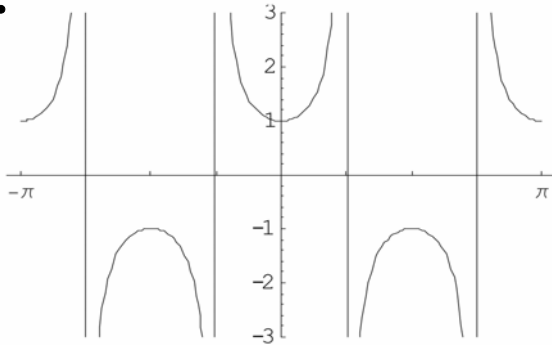
$$x = \frac{\pi}{2} \pm n\pi$$

Trigonometry Lesson 9

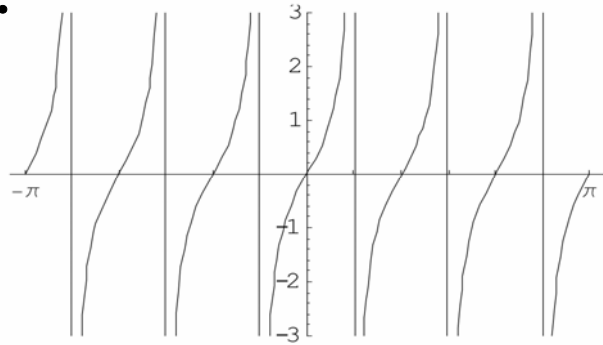
Part I - Graphing Other Trig Functions

Find the equation of each of the following graphs. Also, state the general solution of the asymptotes.

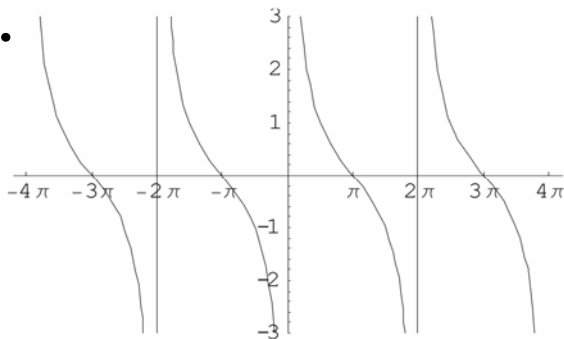
1.



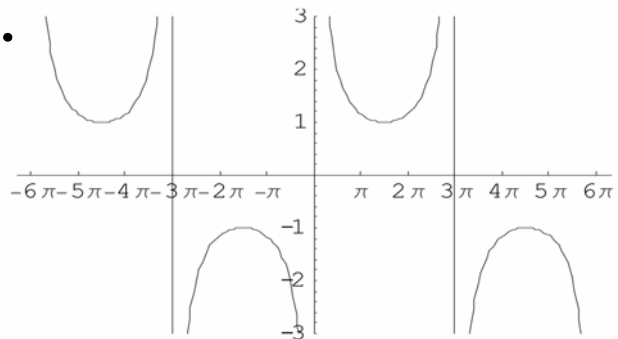
2.



3.

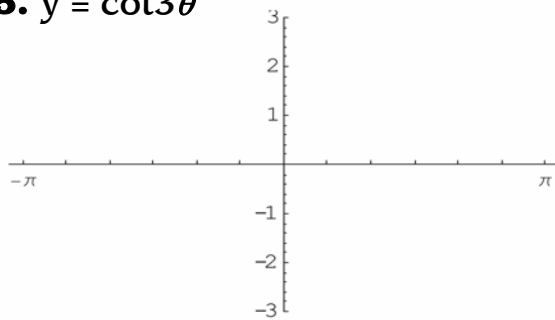


4.

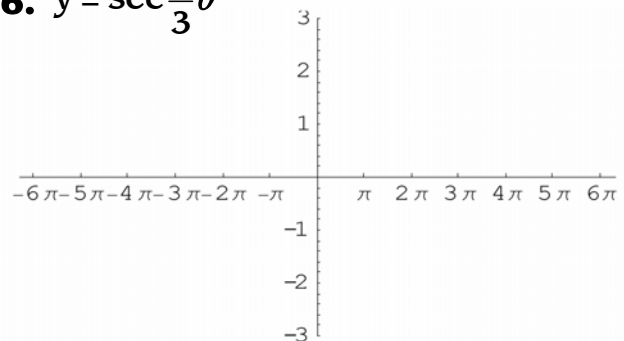


Draw the graph for each of the following equations, and state the general solution of the asymptotes.

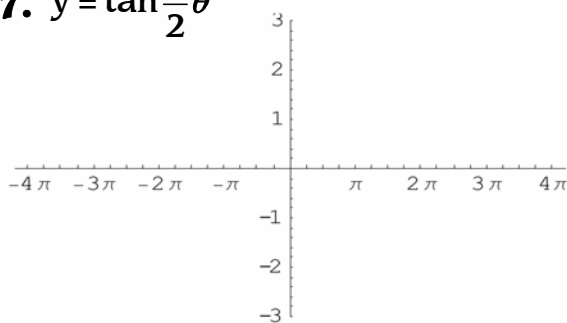
5. $y = \cot 3\theta$



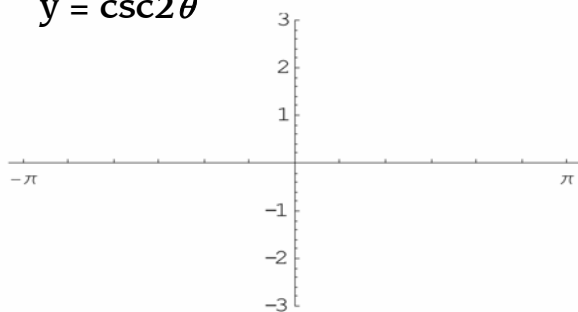
6. $y = \sec \frac{1}{3}\theta$



7. $y = \tan \frac{1}{2}\theta$



8. $y = \csc 2\theta$



Trigonometry Lesson 9

Part I - Graphing Other Trig Functions

1) The period of this $\sec\theta$ graph is π .

(The length of one complete cycle must contain both the top U and the upside-down U.)

Now find the b-value: $b = \frac{2\pi}{\text{Period}} = \frac{2\pi}{\pi} = 2$. Equation is: $y = \sec 2\theta$.

General Solution: $x = \frac{\pi}{4} \pm n \frac{\pi}{2}$

2) The period of this $\tan\theta$ graph is $\pi/3$. We can tell since each tick is 30° and one cycle uses two ticks.

Now find the b-value: $b = \frac{\pi}{\text{Period}} = \frac{\pi}{\frac{\pi}{3}} = \pi \times \frac{3}{\pi} = 3$. Equation is $y = \tan 3\theta$.

General Solution: $x = \frac{\pi}{6} \pm n \left(\frac{\pi}{3}\right)$

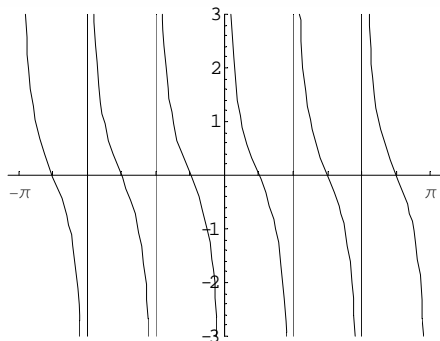
3) The period of the $\cot\theta$ graph is 2π . Now find the b-value: $b = \frac{\pi}{\text{Period}} = \frac{\pi}{2\pi} = \frac{1}{2}$ Equation is $y = \cot \frac{1}{2}\theta$

General Solution: $x = \pm n(2\pi)$

4) The period of the $\csc\theta$ graph is 6π . Now find the b-value: $b = \frac{2\pi}{\text{Period}} = \frac{2\pi}{6\pi} = \frac{1}{3}$ Equation is $y = \csc \frac{1}{3}\theta$

General Solution: $x = \pm n(3\pi)$

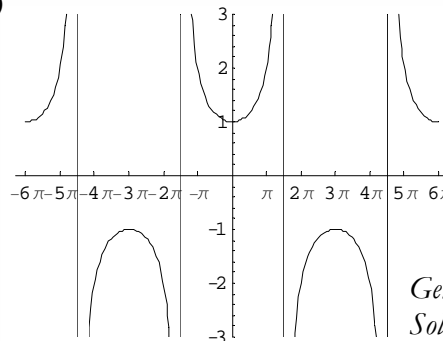
5)



General Solution:

$$x = \pm n \left(\frac{\pi}{3}\right)$$

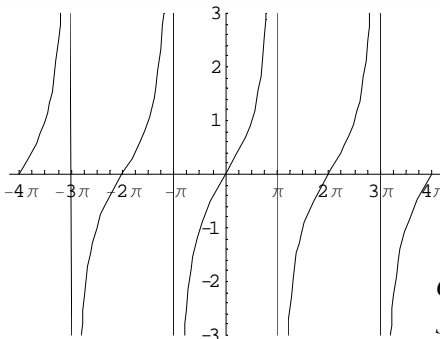
6)



General Solution:

$$x = \frac{3\pi}{2} \pm n(3\pi)$$

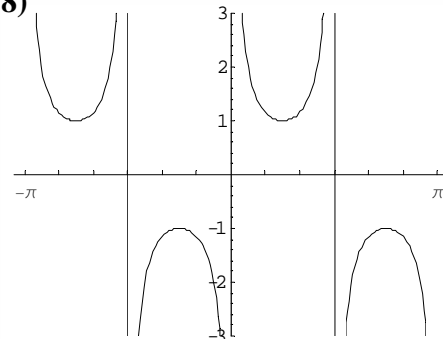
7)



General Solution:

$$x = \pi \pm n(2\pi)$$

8)



General Solution:

$$x = \pm \frac{n\pi}{2}$$

Trigonometry Lesson 9

Part II - Summary of Trig Functions

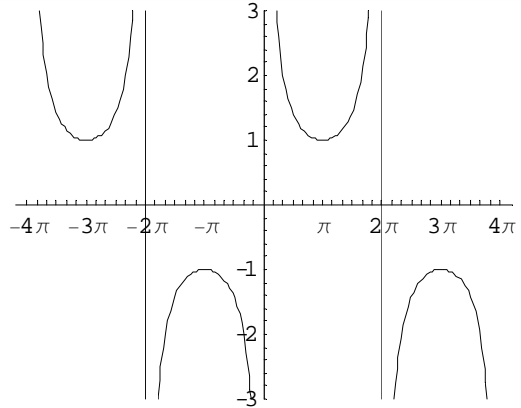
Example 1: Given the equation: $y = \csc \frac{1}{2}\theta$, find the following:

Questions:

- a) a-value
- b) b-value
- c) Period
- d) Phase shift
- e) Vertical Displacement
- f) Domain
- g) Range
- h) x-intercepts
- i) y-intercepts
- j) Equation of asymptotes

Answers:

- a) 1
- b) $\frac{1}{2}$
- c) $Period = \frac{2\pi}{b} = \frac{2\pi}{0.5} = 4\pi$
- d) None
- e) None
- f) $x \neq \pm n(2\pi)$
- g) $y \leq -1, y \geq 1$
- h) None
- i) None
- j) $x = \pm n(2\pi)$



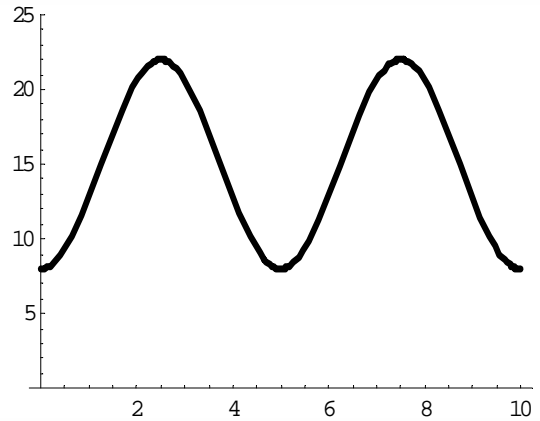
Example 2: Given the equation: $h(t) = 7\sin \frac{2\pi}{5}(t - 1.25) + 15$, find the following:

Questions:

- a) a-value
- b) b-value
- c) Period
- d) Phase shift
- e) Vertical Displacement
- f) Domain
- g) Range
- h) x-intercepts
- i) y-intercepts
- j) Equation of asymptotes

Answers:

- a) 7
- b) $\frac{2\pi}{5}$
- c) 5
- d) 1.25 right
- e) 15 up
- f) $t \in R$
- g) $8 \leq h(t) \leq 22$
- h) None
- i) 8
- j) None



Reminder: To find x-intercepts, use
 $2^{nd} \rightarrow Trace \rightarrow Zero$.
 You should always state general solutions for x-intercepts.
(unless the domain is specified)

Find y-intercepts using:
 $2^{nd} \rightarrow Trace \rightarrow Value \rightarrow x=0$

Trigonometry Lesson 9

Part II - Summary of Trig Functions

	a-value	b-value	Period	Phase Shift	Vertical Displacement	Domain	Range	x-int	y-int	Equation Of Asymptotes
1.	2	4	$\frac{\pi}{2}$	$\frac{\pi}{4}$ right	5 up	$\theta \in R$	$3 \leq y \leq 7$	None	5	None
2.	21	$\frac{2\pi}{365}$	365	118 right	5.5 up	$d \in R$	$-15.5 \leq T \leq 26.5$	$102^\circ \pm n(360^\circ)$ $316^\circ \pm n(360^\circ)$	-13.31	None
3.	1	$\frac{1}{2}$	2π	None	None	$x \neq \pi \pm n(2\pi)$	$y \in R$	$\pm n(2\pi)$	0	$x = \pi \pm n(2\pi)$
4.	$\frac{1}{2}$	2	π	45° right	1 up	$\theta \in R$	$0.5 \leq y \leq 1.5$	None	1	None
5.	18.5	$\frac{2\pi}{365}$	365	28 right	4.5 up	$t \in R$	$-14 \leq h \leq 23$	$134^\circ \pm n(360^\circ)$ $303^\circ \pm n(360^\circ)$	20.9	None
6.	$\frac{1}{2}$	$\frac{1}{3}$	6π	$\frac{\pi}{2}$ left	3 up	$\theta \in R$	$2.5 \leq y \leq 3.5$	None	3.25	None
7.	1	$\frac{1}{3}$	6π	None	None	$x \neq \frac{3\pi}{2} \pm n(3\pi)$	$y \leq -1$ & $y \geq 1$	None	1	$x = \frac{3\pi}{2} \pm n(3\pi)$
8.	1	$\frac{1}{2}$	2π	None	None	$x \neq \pm n(2\pi)$	$y \in R$	$\pi \pm n(2\pi)$	None	$x = \pm n(2\pi)$
9.	2	$\frac{1}{3}$	6π	π right	2 up	$\theta \in R$	$0 \leq y \leq 4$	None	0.29	None
10	20.1	$\frac{2\pi}{300}$	300	265 right	6.2 up	$t \in R$	$-13.9 \leq y \leq 26.3$	$130^\circ \pm n(360^\circ)$ $250^\circ \pm n(360^\circ)$	19.6	None