Pure Math 30

EXPLAINED!

Diploma Style Practice Exam

Perms and Combs

- Answers -

Perms and Combs Practice Exam - ANSWERS

These are the formulas for Perms & Combs you will be given on your diploma

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

$$ {}_nC_r = \frac{n!}{(n-r)!r!} $$

$$ t_{k+1} = {}_nC_k x^{n-k} y^k $$

ANSWERS

1. D

2. D

3. B

4. D

5. B

6. C

NR 1. 36

7. B

8. B

NR 2. 10

9. A

10. A

11. A

12. C

NR 3. 5

13. B

14. D

NR 4. 1440

15. A

16. C

17. B

18. C

19. B

20. A

NR 5. 1

NR 6. 9375

21. C

22. B

23. A

24. A

NR 7. 17

25. B

26. A

27. D

28. D

29. B

30. C

31. D

32. B

33. C

1. The number of ways 12 teams can play each other *once* is ${}_{12}C_2$. Since each of these combinations happens twice, multiply the result by 2 to find the number of games in total. The answer is ${}_{12}C_2 \times 2$.
The answer is **D**.

2. Draw the letters of **KITCHEN**, keeping the vowels in a bubble.

I E K T C H N

There are six items, which can be arranged in 6! ways. The vowels can be arranged in 2! ways inside the bubble. Thus, there are 6!•2! = 1440 ways of arranging the letters keeping the vowels together.
The answer is **D**.

3. There are 2 parents who could go on the left end of the line. Once that parent is placed, only one parent remains for the other end of the line. The remaining six people fill out the middle of the line in any order.

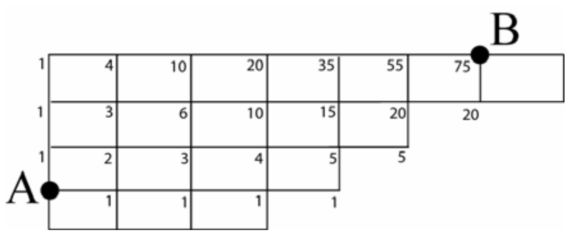
$\underline{2}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \underline{1}$

Multiplying, the answer is 1440.
The answer is **B**.

4. In the first cube, the possible directions are up, right, and back. Writing as **URB**, this set of letters can be arranged in 3! = 6 ways. Since the second cube can be written as **URB** as well, it has 6 possible paths too. To combine the pathways, multiply the results. $6 \times 6 = 36$ possible paths.
The answer is **D**.

5. There are ten people, and we want six. If Kirsten and James must be on the committee, that reduces the number of available people to 8, and the number of positions remaining on the committee to 4.
The number of possible committees is ${}_8C_4$
The answer is **B**.

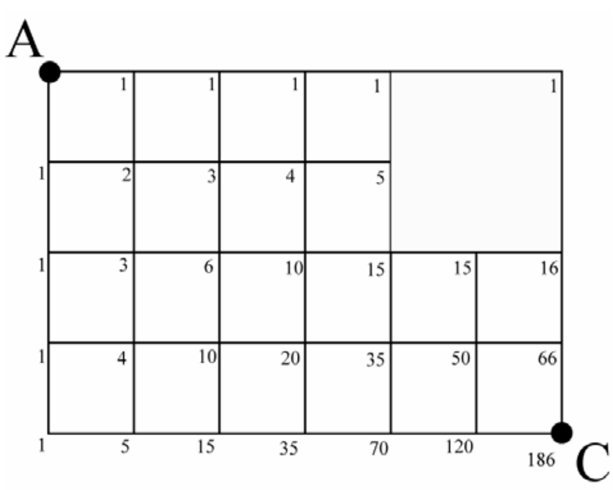
6. Fill in the pathway using Pascal's Triangle.



There are 75 paths.
The answer is **C**.

NR 1) Use combinations since we don't care what order the games are purchased in.
$_4C_3 \times _3C_1 \times _2C_2 \times _3C_1 = 36$
The answer is **36**.

7. Fill in the pathway using Pascal's Triangle. Omit the paths passing through point B.



The answer is **B**.

8. The number of lines that can be drawn from four points on a circle is $_4C_2$
The only option given that has the same solution is the number of ways four people can shake hands once.
The answer is **B**.
(The answer for ***A*** *is* $2^4 - 1$*, the answer for* ***C*** *is* $_6C_2$*, and the answer for* ***D*** *is 5)*

NR 2) In the expansion of $(a+b)^n$, the number of terms is always one greater than the exponent. So, if $\left(3x^2-2y^3\right)^{3k-9}$ has 22 terms, that means the exponent must equal 21.

$3k-9=21$
$3k=30$
$k=10$

The answer is **10**.

9. Use combinations since we don't care what order the committee is.
${}_{10}C_4 \times {}_{13}C_5$
The answer is **A**.

10. Use the formula $t_{k+1} = {}_nC_k x^{n-k} y^k$ to solve this question. First place everything you know into the equation, leaving k blank for now.
$t_{\square+1} = {}_8C_{\square}(mx)^{8-\square}(-4)^{\square}$
By inspection, we get a term containing x^4 when k = 4.
$t_{4+1} = {}_8C_4(mx)^{8-4}(-4)^4$
$t_{4+1} = {}_8C_4(mx)^4(-4)^4$
$t_5 = 17920m^4x^4$
Now plug in the known term in the left side
$1451520x^4 = 17920m^4x^4$
$1451520 = 17920m^4$
$81 = m^4$
$m = 3$
The answer is **A**.

11. The number of arrangements is $\dfrac{33!}{6!\bullet 8!\bullet 10!\bullet 9!} = 2.27\times 10^{17}$
The answer is **A**.

12. The number of ways the pumpkins and watermelons are together is 4! • 2! = 48
The number of arrangements without restrictions is 5! = 120

The number of ways pumpkins and watermelons are NOT together is 120 – 48 = 72
The answer is **C**.

NR 3) Write ${}_nP_r = 6720$ as $\dfrac{n!}{(n-r)!} = 6720$

Write ${}_nC_r = 56$ as $\dfrac{n!}{(n-r)!r!} = 56$

Now divide the expressions to simplify

$$\frac{\dfrac{n!}{(n-r)!}}{\dfrac{n!}{(n-r)!r!}} = \frac{n!}{(n-r)!} \times \frac{(n-r)!r!}{n!} = r!$$

On the right side, $\dfrac{6720}{56} = 120$

$r! = 120$ when $r = 5$.
The answer is **5**.

13. Use the formula $t_{k+1} = {}_nC_k x^{n-k} y^k$ to solve this question. First place everything you know into the equation, leaving k blank for now.

$$t_{\square+1} = {}_3C_{\square}\left(2x^2\right)^{3-\square}\left(\frac{m}{y}\right)^{\square}$$

By inspection, we get a term containing $\dfrac{x^2}{y^2}$ when $k = 2$.

$$t_{2+1} = {}_3C_2\left(2x^2\right)^{3-2}\left(\frac{m}{y}\right)^2$$

$$t_{2+1} = {}_3C_2\left(2x^2\right)\left(\frac{m}{y}\right)^2$$

$$t_3 = \frac{6x^2m^2}{y^2}$$

Now plug in the known term in the left side

$$\frac{54x^2}{y^2} = \frac{6x^2m^2}{y^2}$$

$$54 = 6m^2$$

$$9 = m^2$$

$$m = 3$$

The answer is **B**.

14. Write out the letters representing the possible directions. **LLLLDDDDFFFFF**

The arrangements are $\dfrac{13!}{4!\bullet 4!\bullet 5!} = 90090$

The answer is **D**.

NR 4) Write out the vowels in a bubble, and the other letters on the side.

There are six items, which can be arranged in 6! ways. The vowels can be arranged in 2! ways inside the bubble. Thus, there are 6!•2! = 1440 ways of arranging the letters keeping the vowels together.

The answer is **1440**.

15. Determine the number of possible arrangements for a three-digit even number, a four-digit even number, then add the results together.

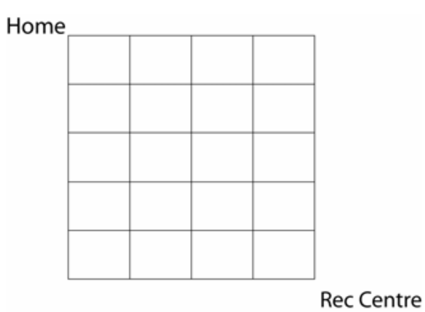
4 3 2 + 4 3 2 2

Fill in a 2 in the last space since there are two even numbers to choose from, then fill in the rest of the spaces with the remaining digits.

The answer is 72

The answer is **A**.

16. Draw out the map



There are 4 blocks East, then 5 blocks South. EEEESSSSS

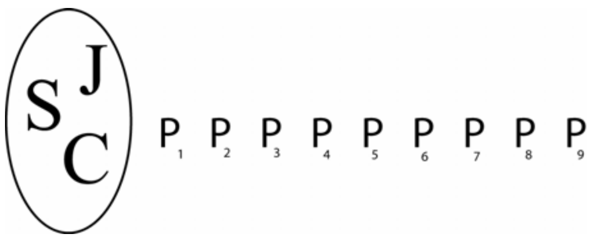
$\dfrac{9!}{4!\bullet 5!} = 126$

The answer is **C**.

17. Since we are drawing specific people from specific selection pools, we can use combinations.
Out of four accountants, we need one.
Out of three marketing agents, we need two.
Out of seven board members, we need four.
${}_4C_1 \times {}_3C_2 \times {}_7C_4 = 420$
The answer is **B**.

18. Each bubble has two options. It's either shaded in, or it's not.
$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$
The answer is **C**.

19. Put Crystal, Steven, and Jason in a bubble, and write out the nine other people.



There are 10 items, which can be arranged in 10! ways. Crystal, Steven, and Jason can be arranged in 3! ways inside the bubble. Thus, there are 10!•3! ways of arranging the people in the line.
The answer is **B**.

20. We can arrange the first four blocks in ${}_{26}P_4$ ways, then the next three blocks in ${}_{22}P_3$ ways since only 22 blocks remain. Then we can arrange the next two in ${}_{19}P_2$ ways, and the top block can be picked in ${}_{17}P_1$ ways. Since we are forming a single group (a pyramid) we multiply all the cases together.
The answer is **A**.

NR 5) The error is in Step 1, since the formula for a permutation was used instead of a combination.
The answer is **1**.

NR 6) Old code: $5\times5\times5\times5\times5\times5 = 15625$
New code: $5\times5\times10\times10\times10 = 25000$

The increase in the number of codes is **9375**

21. We can treat this as arranging 15 items, eliminating identical cases.

The answer is $\frac{15!}{4!\times 3!\times 6!\times 2!}$

The answer is **C**.

22. If there are six points on a circle, we need to select three to make a triangle.

This can be done in ${}_6C_3 = 20$ ways.

The answer is **B**.

23. Use the formula $t_{k+1} = {}_nC_k x^{n-k} y^k$ to solve this question. First place everything you know into the equation, leaving k blank for now.

$t_{\square+1} = {}_7C_{\square}\left(x^3\right)^{7-\square}\left(-4\right)^{\square}$

By inspection, we get a term containing x^{12} when $k = 3$.

$t_{3+1} = {}_7C_3\left(x^3\right)^{7-3}\left(-4\right)^3$

$t_4 = {}_7C_3\left(x^3\right)^4\left(-4\right)^3$

$t_4 = -2240x^{12}$

The coefficient is -2240

The answer is **A**.

24. Using the formula for handshakes, we get ${}_nC_2 = 91$. The easiest way to do this question is to use trial and error in your calculator. Doing so gives a result of 14 for n.

The answer is **A**.

NR 7) Positions equidistant from the ends of Pascal's triangle have the same value. So, if there are 20 positions, three from the front yields the same value as three from the back, so 17 will give the same result.

The answer is **17**.

25. We could use the term formula $t_{k+1} = {}_nC_k x^{n-k} y^k$ to get each coefficient, then add them up. A shortcut would be to recognize that the coefficients in the expansion of $\left(x-1\right)^n$ follow Pascal's Triangle, with alternating signs.

This gives 1 – 6 + 15 – 20 + 15 – 6 + 1 = 0

The answer is **B**.

26. There are three objects (**XYZ**) and the possible arrangements are shown.
The answer is **A**.

27. The total number of committees is ${}_{15}C_6 = 5005$
The total number of committees with **no man** is ${}_{9}C_6 = 84$
Total cases – Unwanted cases = 5005 – 84 = 4921
The answer is **D**.

28. The first car to park has 6 options. The second car to park has 5 options. The third car has 4 options. The fourth car has 3 options. The fifth car has 2 options.
$6 \times 5 \times 4 \times 3 \times 2 = 720$
The answer is **D**.

29. The first set of colored balls can be chosen in ${}_4C_2 = 6$ ways.
The second set of colored balls can be chosen in ${}_4C_2 = 6$ ways.
Multiplying (since we are forming a single group from different selection pools) gives the result $6 \times 6 = 36$.
The answer is **B**.

30. Choose the vowels in ${}_3C_2 = 3$ ways
Choose the consonants in ${}_5C_2 = 10$ ways
Now arrange the letters in 4! = 24 ways.
${}_3C_2 \times {}_5C_2 \times 4! = 720$
The answer is **C**.

31. ${}_aC_3$ has *a* terms, and ${}_aC_3$ represents the value of the third term in.
${}_aC_{a-3}$ has *a* terms, and ${}_aC_{a-3}$ represents the value of the third term back.
Terms in Pascal's Triangle the same distance in from each end of the row have the same value.
The answer is **D**.

32. This is a combination since the order does not matter. The child can select zero toys, one toy, two toys, or three toys. Since every case is independent of each other, add the results. The answer is ${}_8C_0 + {}_8C_1 + {}_8C_2 + {}_8C_3$
The answer is **B**.

33. At least 5 chemists means there can be (5 chemists / 1 other profession) + (6 chemists). Writing this out using combinations, we get
${}_{10}C_5 \bullet {}_{28}C_1 + {}_{10}C_6 = 7266$
The answer is **C**.

Written Response 1:

- Suppose there are five solvents (**V,W,X,Y,Z**). These solvents can be arranged in 5! = 120 ways. Now suppose we remove solvent Z, but still want to arrange the remaining four solvents in five flasks. Call the new set (**V,W,X,Y,E**), where E represents the empty flask. We can arrange these five letters in 5! = 120 ways.
  (2 Marks)
  (Note that simply saying "because $_5P_4 = 120$ *" is zero marks)*

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Number of Solvents	**Number of Arrangements in Five Beakers**
0	1
1	5
2	20
3	60
4	120
5	120

(2 Marks)

To arrange 3 solvents, we have the set (**V,W,X,E,E**), which can be arranged in $\frac{5!}{2!} = 60$ ways.

To arrange 2 solvents, we have the set (**V,W,E,E,E**), which can be arranged in $\frac{5!}{3!} = 20$ ways.

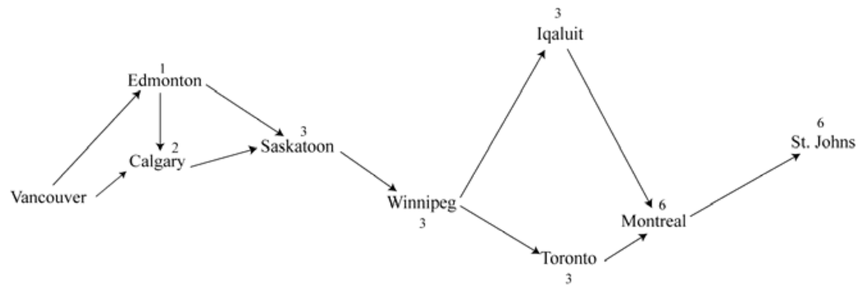
To arrange 1 solvents, we have the set (**V,E,E,E,E**), which can be arranged in $\frac{5!}{4!} = 5$ ways.

To arrange 0 solvents, we have the set (**E,E,E,E,E**), which can be arranged in $\frac{5!}{5!} = 1$ ways.

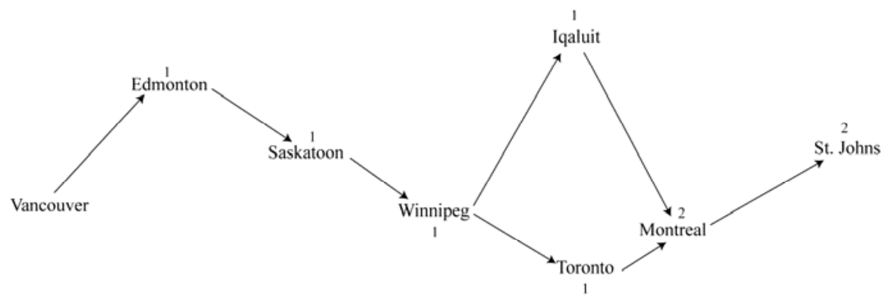
- By looking at the results in calculating the table values, the pattern can be observed as $\frac{m!}{(m-k)!}$, where *m* is the number of flasks, and *k* is the number of solvents.
  (2 Marks)

Written Response 2:

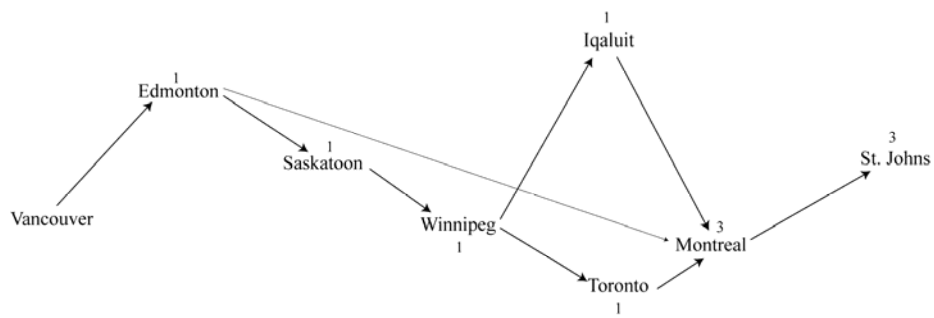
- Use Pascal's Triangle to determine the number of paths.



There are 6 paths. **(2 Marks)**

- 

There are 2 paths. **(1 Mark)**

- 

There are 3 paths. **(1 Mark)**

- Order is important in a touring schedule, so use permutations.
  Out of 12 possible cities, the band wants to create a schedule with only 5.
  ${}_{12}P_5 = 95040$ **(1 Mark)**

- The first five cities in Southern AB can be ordered in ${}_5P_5$ ways. The two cities in Central AB can be ordered in ${}_4P_2$ ways. The final cities in Northern AB can be ordered in ${}_3P_2$ ways. ${}_5P_5 \times {}_4P_2 \times {}_3P_2 = 8640$ **(1 Mark)**

- Remove Banff & Fairview from the selection pool. The first five cities in Southern AB can now be ordered in ${}_4P_4$ ways. The two cities in Central AB can be ordered in ${}_4P_2$ ways. The final cities in Northern AB can be ordered in ${}_2P_2$ ways. ${}_4P_4 \times {}_4P_2 \times {}_2P_2 = 576$ **(1 Mark)**

Written Response 3:

- A person can go on all six slides in 6! = 720 ways. **(1 Mark)**

- Each bubble has only two options: it's filled in, or it's not. That means there are $2\times2\times2\times2\times2\times2=64$ different answer sheets that can be created. **(1 Mark)**

- If the person must go on at least one slide, then the only case not allowed is when the card has no bubbles filled in. Total cases – Unwanted cases = 64 – 1 = 63.
  (1 Mark)
  (*Alternative: Use the one or more formula* $2^n - 1 = 2^6 - 1 = 63$)

- Use a combination since we are only selecting the rides, not ordering them.
  $${}_nC_2 = 36$$
  $$\frac{n!}{(n-2)!2!} = 36$$
  $$\frac{n!}{(n-2)!2} = 36$$
  $$n! = 72(n-2)!$$
  $$n(n-1)(n-2)! = 72(n-2)!$$
  $$n(n-1) = 72$$
  $$n^2 - n - 72 = 0$$
  $$(n-9)(n+8) = 0$$
  $$n = 9$$
  (2 Marks)

- $\frac{n!}{(n-r)!}$ can also be written as ${}_nP_r$. An example of a question would be:

  "If there are *n* waterslides, how many ways can a person select the order they want to go on *r* of those slides?" **(1 Mark)**