



Pure Math 30

EXPLAINED!

*Diploma Style
Practice Exam*

Perms and Combs
- QUESTIONS -

Formulas

These are the formulas for Perms & Combs you will be given on your diploma

$${}_n P_r = \frac{n!}{(n-r)!}$$

$${}_n C_r = \frac{n!}{(n-r)!r!}$$

$$t_{k+1} = {}_n C_k x^{n-k} y^k$$

<i>Use this sheet to record your answers</i>
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- | | | | |
|-------|-------|-------|-----|
| 1. | 10. | 19. | 27. |
| 2. | 11. | 20. | 28. |
| 3. | 12. | NR 5. | 29. |
| 4. | NR 3. | NR 6. | 30. |
| 5. | 13. | 21. | 31. |
| 6. | 14. | 22. | 32. |
| NR 1. | NR 4. | 23. | 33. |
| 7. | 15. | 24. | |
| 8. | 16. | NR 7. | |
| NR 2. | 17. | 25. | |
| 9. | 18. | 26. | |

Perms & Combs Diploma Style Practice Exam

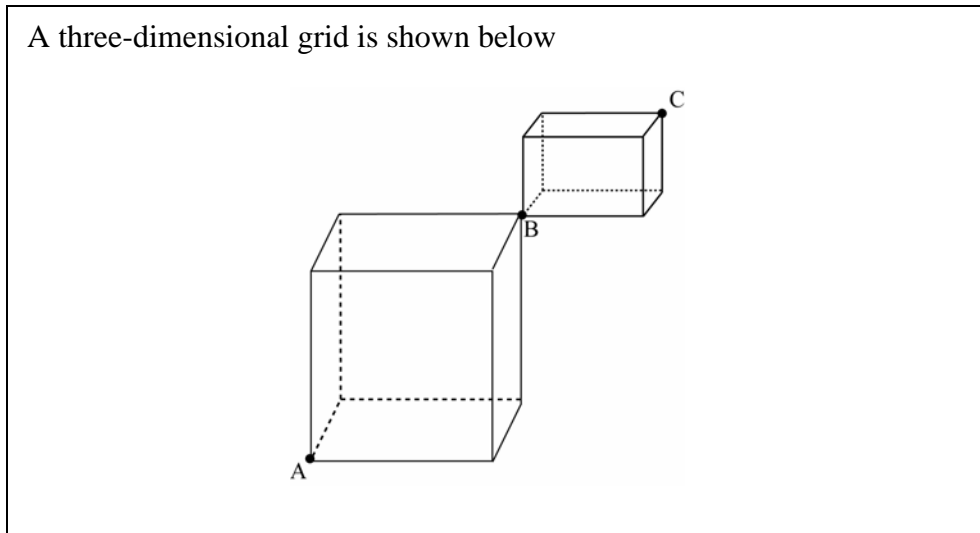
Use the following information to answer the next question.

There are 12 teams in a soccer league, and each team must play each other *twice* in a tournament.



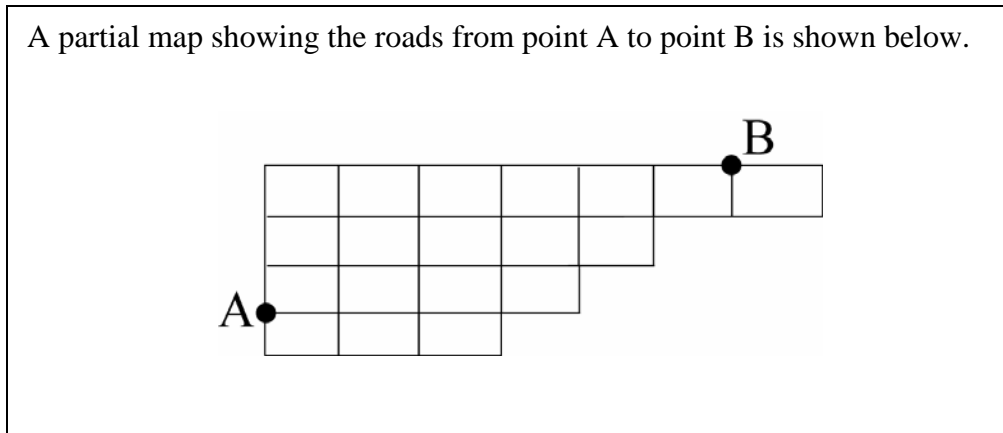
- The number of games that will be played in total is
 - ${}_6P_2 \times 2$
 - ${}_{12}C_2$
 - ${}_{12}P_2 \times 2$
 - ${}_{12}C_2 \times 2$
- The number of distinguishable arrangements that can be made from the word **KITCHEN**, if the vowels must stay together, is
 - $2! \times 5! \times 2!$
 - $2! \times 5!$
 - ${}_7P_2 \times {}_5P_5$
 - $2! \times 6!$
- A family is being arranged in a line for a group photograph. If the family consists of a mother, a father, a baby, and five children, the number of arrangements that begin and end with a parent is
 - 720
 - 1440
 - 5040
 - 40320

Use the following information to answer the next question.



4. The number of pathways from A to C, passing through B, is
- A. 6
 - B. 12
 - C. 18
 - D. 36
5. There are ten people available for appointment to a committee consisting of six people. The number of committees that can be formed, if Kirsten and James must be on the committee, is
- A. ${}_8P_4$
 - B. ${}_8C_4$
 - C. ${}_{10}P_6 - {}_2P_2$
 - D. ${}_{10}C_6 - {}_2C_2$

Use the following information to answer the next question.



6. If the only allowed directions are North and East, the number of pathways from point A to point B is
- A. 30
 - B. 60
 - C. 75
 - D. 90

Use the following information to answer the next question.

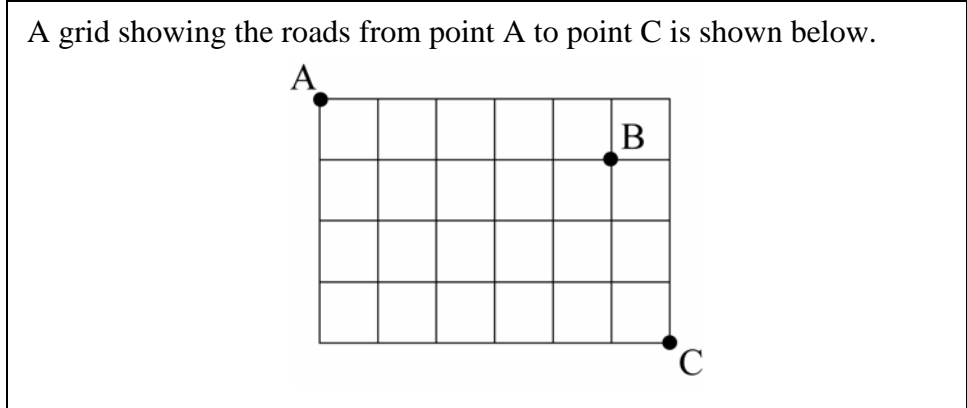
A used video game store has the following titles available for purchase

Action	RPG	Sports	Classic
Legally Owned Auto	Final Quest MCCXXI	Minor League Baseball 2K6	Dot-Gobbler
Crypt Infiltrator	Old Scrolls IV	Smashdown 2007	Punky the Porcupine
Angelic Circle III	Neverautumn Days		Falling Blocks
Warfield II			

Numerical Response

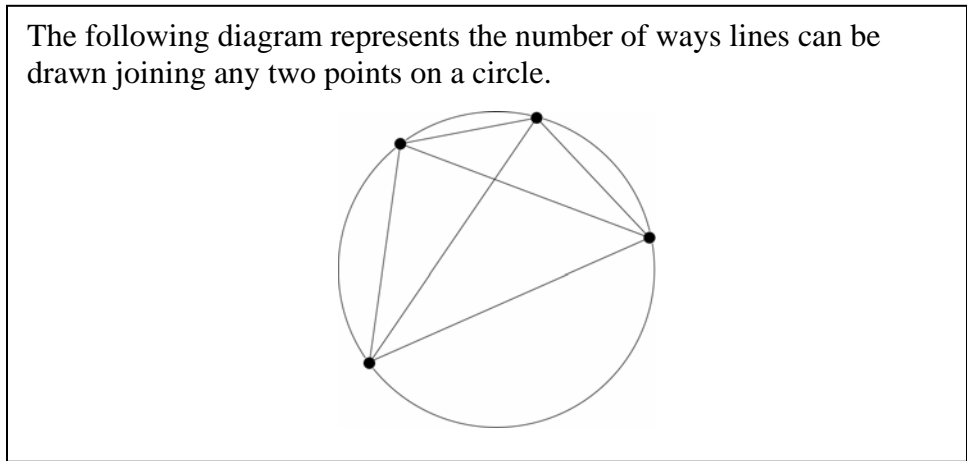
1. If a customer purchases 3 action games, 1 RPG, 2 sports, and 1 classic game, the total number of ways he can select the games is _____ .

Use the following information to answer the next question.



7. If the only allowed directions are South and East, the number of pathways from point A to point C that do **not** pass through B is
- A. 141
 - B. 186
 - C. 206
 - D. 210

Use the following information to answer the next question.



8. A situation that could be modeled using the above diagram is
- A. the number of ways to choose at least one of four toys
 - B. the number of ways four people can shake hands once
 - C. the number of ways a two person team can be formed from six people
 - D. the number of terms in $(x + y)^4$

Numerical Response

2. The expansion of $(3x^2 - 2y^3)^{3k-9}$ has 22 terms. The value of k is, to the nearest whole number, _____.
9. The number of committees consisting of 4 men and 5 women that can be formed from 10 men and 13 women is
- A. ${}_{10}C_4 \times {}_{13}C_5$
B. ${}_{10}P_4 \times {}_{13}P_5$
C. ${}_{23}C_9$
D. ${}_{23}P_9$
10. A term in the expansion of $(mx - 4)^8$ is $1451520x^4$. The value of m is
- A. 3
B. 5
C. 6
D. 9

Use the following information to answer the next question.

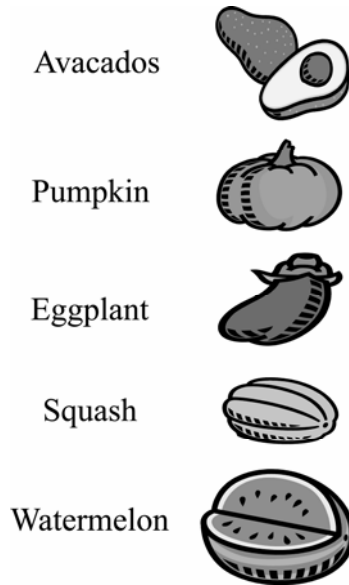
The *Trigonometry I Diploma Style Practice Exam* has 6 questions with the answer **A**, 8 questions with the answer **B**, 10 questions with the answer **C**, and 9 questions with the answer **D**



11. The number of different answer keys that can be created with the letters above is
- A. 2.27×10^{17}
B. 3.82×10^{19}
C. 4.00×10^{24}
D. 8.68×10^{36}

Use the following information to answer the next question.

A grocery store manager is trying to decide how to arrange the following items in the produce aisle



12. The items can be arranged in any order, but the manager does not want the pumpkins and watermelons together. If this is the only restriction, the number of possible arrangements is
- A. 5
 - B. 45
 - C. 72
 - D. 120

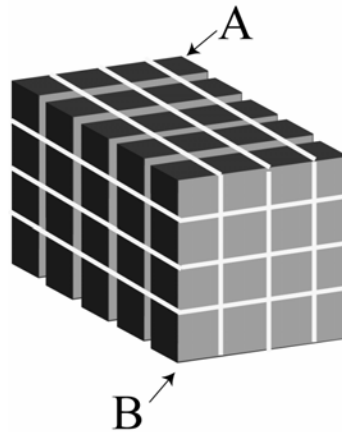
Numerical Response

3. If ${}_n P_r = 6720$ and ${}_n C_r = 56$, then the value of r is _____ .

13. If a term in the expansion of $\left(2x^2 + \frac{m}{y}\right)^3$ is $\frac{54x^2}{y^2}$, the value of m is
- A. 2
 - B. 3
 - C. 4
 - D. 5

Use the following information to answer the next question.

A student wishes to determine the number of paths from point A to point B in a set of stacked blocks. The path must go left, forward, and down only, with no backtracking. (*Paths within the shape are allowed.*)



14. The number of paths from point A to point B is
- A. 12043
 - B. 32800
 - C. 67000
 - D. 90090

Numerical Response

4. If all of the letters in the word **PENCILS** are used, the number of arrangements with all the vowels together is _____.

15. The number of three digit **or** four digit even numbers that can be formed from the numbers 2, 3, 5, 6, 7 is
- A. 72
 - B. 120
 - C. 144
 - D. 5040
16. The map of a small town has streets drawn vertically, and avenues drawn horizontally. A student wishes to walk to the recreation centre, which is 4 blocks East and 5 blocks South of his home. The number of different routes to the recreation centre that are 9 blocks in length are
- A. 35
 - B. 111
 - C. 126
 - D. 362880
17. A committee requires one accountant, two marketing agents, and four board members. If there are four accountants, three marketing agents, and seven board members available for selection in the committee, the number of committees that can be formed is
- A. 42
 - B. 420
 - C. 20160
 - D. ${}_{14}P_7$

Use the following information to answer the next question.

A questionnaire asks students what activities they enjoy. It is filled out by shading in the circle beside each activity a person enjoys, and a student may select no activity, one activity, or more than one activity.

- Mountain Climbing
- Canoeing
- Video Games
- Bowling
- Billiards
- Walking
- Hockey

18. The number of ways a student could fill out the questionnaire is
- A. 21
 - B. 42
 - C. 128
 - D. 5040
19. There are 12 people in line for a movie. If Crystal, Steven, and Jason are friends and will always stand together, the total number of possible arrangements for the entire line is
- A. $3! \times 2! \times 9!$
 - B. $3! \times 10!$
 - C. ${}_{12}P_3 \times 9!$
 - D. ${}_{12}C_3 \times 9!$

Use the following information to answer the next question.

A child has a set of 26 alphabetized blocks, and stacks 10 of these blocks in the shape of a pyramid.



20. The number of possible 10-block pyramids that can be formed from the entire set of 26 blocks is

- A. ${}_{26}P_4 \cdot {}_{22}P_3 \cdot {}_{19}P_2 \cdot {}_{17}P_1$
 B. ${}_{26}P_4 + {}_{22}P_3 + {}_{19}P_2 + {}_{17}P_1$
 C. ${}_{26}C_4 \cdot {}_{22}C_3 \cdot {}_{19}C_2 \cdot {}_{17}C_1$
 D. ${}_{26}C_4 + {}_{22}C_3 + {}_{19}C_2 + {}_{17}C_1$

Use the following information to answer the next question.

A student solves the equation ${}_n C_2 = 10$ using the following steps, but finds the solution yields decimal answers and therefore must not be correct.

Step 1: $\frac{n!}{(n-2)!} = 10$

Step 2: $n! = 10(n-2)!$

Step 3: $n(n-1)(n-2)! = 10(n-2)!$

Step 4: $n(n-1) = 10$

Step 5: $n^2 - n - 10 = 0$

Numerical Response

5. The student made an error in step _____.

Numerical Response

6. A security code used to consist of two odd digits, followed by four even digits. To allow more codes to be generated, a new system uses two even digits, followed by any three digits. If repeated digits are allowed, the **increase** in the number of possible codes is _____.
21. A multiple choice test has 15 questions. Four of these questions have A as an answer, three have B as an answer, six have C as an answer, and two have D as an answer. The number of different answer sheets that can be created is
- A. ${}_{15}C_4 \times {}_{11}C_3 \times {}_8C_6 \times {}_2C_2$
B. ${}_{15}P_4 \times {}_{11}P_3 \times {}_8P_6 \times {}_2P_2$
C. $\frac{15!}{4! \times 3! \times 6! \times 2!}$
D. $15!$
22. Six points are drawn on a circle. The number of triangles that can be formed from these six points is
- A. 10
B. 20
C. 30
D. 720
23. In the expansion of $(x^3 - 4)^7$, the coefficient of x^{12} is
- A. -2240
B. 2240
C. -8960
D. 8960
24. At a business meeting, every person shakes each others hand once. If there were 91 handshakes in total, the number of people at the meeting is
- A. 14
B. 15
C. 24
D. 46

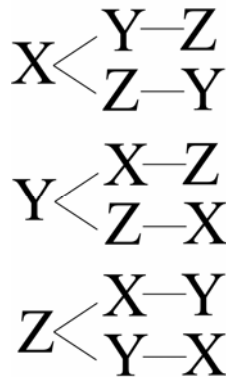
Numerical Response

7. The number of ways 3 tiles can be pulled out of a bag containing 20 tiles is the same as the number of ways k tiles can be pulled out of 20 tiles.
The value of k is _____.

25. If $(x-1)^6 = m_0x^6 + m_1x^5 + m_2x^4 + m_3x^3 + m_4x^2 + m_5x + m_6$, the value of $m_0 + m_1 + m_2 + m_3 + m_4 + m_5 + m_6$ is
- A. -64
B. 0
C. 1
D. 64

Use the following information to answer the next question.

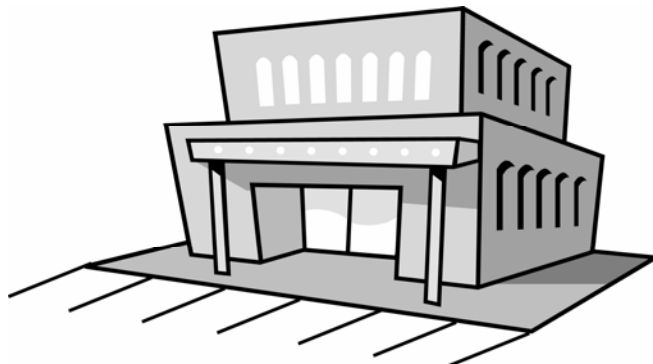
The diagram shown below was drawn by a student to answer a question



26. The above diagram is a solution to the question
- A. What is the total number of permutations of three items?
B. What is the total number of combinations of three items?
C. What is the total number of permutations of six items?
D. What is the total number of combinations of six items?
27. There are 6 men and 9 women available for selection on a 6-person committee. If the committee must have at least one man, the number of possible committees is
- A. 2264
B. 2459
C. 3580
D. 4921

Use the following information to answer the next question.

A building has six parking stalls in front, as shown below.



28. The number of ways five different cars can park in the stalls is
- A. 6
 - B. 56
 - C. 120
 - D. 720
29. One bag contains 4 colored marbles, and another bag contains 4 colored marbles. None of the 8 marbles are the same color. If a person reaches into the first bag and pulls out two marbles, then reaches into the second bag and pulls out two marbles, the number of possible color combinations is
- A. 6
 - B. 36
 - C. 70
 - D. 140

30. A student has 8 tiles that spell the word **COMPUTER**. If the student now wishes to use some of these tiles to make a four-letter word that contains exactly 2 vowels and exactly 2 consonants, the number of possible words is
- A. 335
 B. 480
 C. 720
 D. 1024
31. ${}_a C_3$ is equivalent to
- A. ${}_a C_{3-a}$
 B. ${}_{a-3} C_3$
 C. ${}_3 C_a$
 D. ${}_a C_{a-3}$
32. A child going on a trip is told that out of his 8 favorite toys, he can bring **at most** three toys. The number of ways he could select which toys he brings is
- A. ${}_8 P_0 + {}_8 P_1 + {}_8 P_2 + {}_8 P_3$
 B. ${}_8 C_0 + {}_8 C_1 + {}_8 C_2 + {}_8 C_3$
 C. ${}_8 C_3 - ({}_8 C_0 + {}_8 C_1 + {}_8 C_2)$
 D. ${}_8 C_0 \times {}_8 C_1 \times {}_8 C_2 \times {}_8 C_3$
33. A research team of 6 people is to be formed from 10 chemists, 5 politicians, 8 economists, and 15 biologists. The number of possible teams that can be formed with at least 5 chemists is
- A. 6772
 B. 6934
 C. 7266
 D. 8123

Use the following information to answer the next question.

A chemistry teacher has five flasks on the lab bench labeled A, B, C, D, and E. In preparing a lab for her students, she will randomly place different solvents in each flask and have students use chemical diagnostic tests to determine what is in each beaker.



If five different solvents are placed in the beakers at random, there are $5! = 120$ different arrangements.

Written Response – 10%

1.

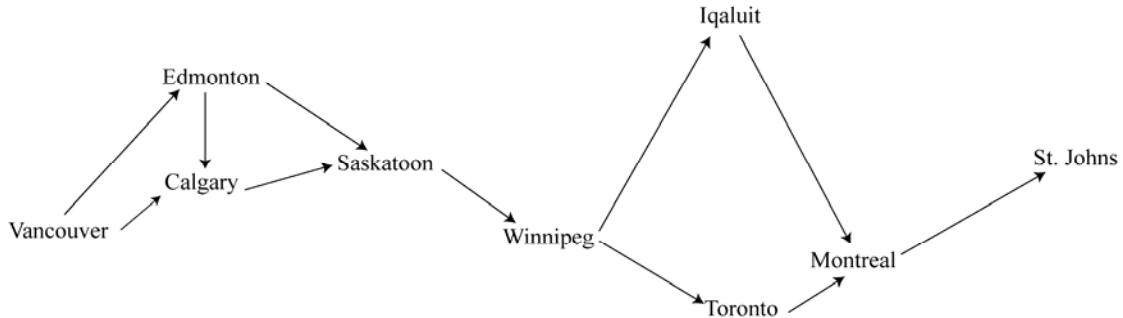
- In an effort to reduce chemical waste, the teacher decides to use only four solvents instead of five. The teacher (who just happens to teach Pure Math 30 as well) does a quick calculation and finds the number of ways to arrange four solvents in five beakers is still 120. Explain this result.
- Complete the following table. Clearly explain how you obtained your answers using the Fundamental Counting Principle.

Number of Solvents	Number of Arrangements in Five Beakers
0	
1	
2	
3	
4	120
5	120

- If there are m flasks available, determine an expression that will give the number ways k solvents can be assigned to m flasks.

Use the following information to answer the next question.

A rock band is planning a tour of Canada, starting in Vancouver and ending in St. John's. The band manager has determined the following routes are possible



Draw lightly in pencil on this map, as you will be using it for the first three parts of this question and may wish to erase your previous work.

Written Response – 10%

2.

- Based on the proposed routes, how many different ways can the band tour Canada?

- The manager decides Calgary, the bands home city, will receive a special concert before the main tour and should be excluded from the proposed routes. How many paths from Vancouver to St. John's are now possible?

- If a direct path from Edmonton to Montreal is now added (with Calgary still off the map), how many routes are now possible?

Use the following additional information to answer the next part of the question.

After completing the national tour, the band returns to Alberta to do an extensive tour of the province. They plan on playing in some of the following cities:

Southern Alberta	Central Alberta	Northern Alberta
Calgary	Edmonton	Grand Prairie
Lethbridge	St. Albert	Fort McMurry
Medicine Hat	Leduc	Fairview
Banff	Vegreville	
Airdrie		

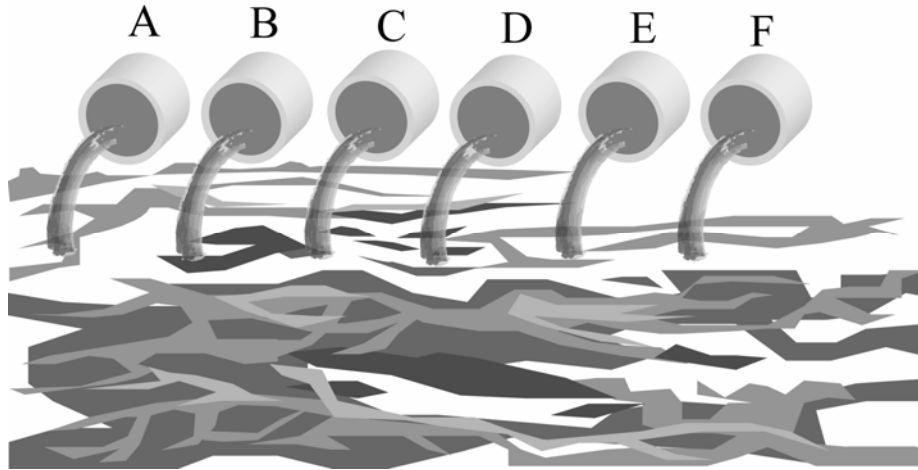
- If the band plays 5 cities in total, how many different concert schedules can be created if there is no restriction on which cities they play?

- If the band must play all cities in Southern Alberta, then two cities in Central Alberta, and finally two cities in Northern Alberta, how many different schedules can be created?

- If the band decides Banff and Fairview should not be on the touring schedule above (*because they intend to record a live album in those cities at a later date*), how many different schedules can be created if the band plays all cities in Southern Alberta, then two cities in Central Alberta, and finally two cities in Northern Alberta?

Use the following information to answer the next question.

There are six waterslides at an amusement park, and the exits to each of these slides are arranged in a line. This arrangement prevents people from colliding with each other while maximizing the available swimming area.



Written Response – 10%

3.

- A person is allowed to go on each slide only once. How many different ways can the order of the slides be chosen if a person goes on all six slides?

- The waterslide attendant keeps track of the slides each person has gone on by filling out the card shown on the right. Filling in a square indicates that particular slide has been used. If a person can choose to go on a slide, or skip the slide, how many different ways can the sheet be filled in?

Waterslide	
A	<input type="checkbox"/>
B	<input type="checkbox"/>
C	<input type="checkbox"/>
D	<input type="checkbox"/>
E	<input type="checkbox"/>
F	<input type="checkbox"/>

- If a person goes on **one or more** of the slides, how many different ways can they select the slides they go on?

- At a different amusement park, there are n slides. If the number of ways a person can select 2 of those slides is 36, **determine algebraically** the number of slides at the amusement park.

- Give an example of a situation (involving waterslides) to which the expression $\frac{n!}{(n-r)!}$ is the solution.