Pure Math 30

EXPLAINED!

Diploma Style Practice Exam

Conics

--ANSWERS--

Conics Diploma Style Practice Exam - ANSWERS

Formulas

These are the only formulas for conics you will be given on your diploma

$Ax^2 + Cy^2 + Dx + Ey + F = 0 \quad , \quad A, C, D, E, F \in I$

$(x-h)^2 + (y-k)^2 = r^2$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$y - k = a(x-h)^2$

$x - h = a(y-k)^2$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = \pm 1$

Answers

1. **A**

NR 1) **49.7**

2. **A**
3. **B**
4. **A**
5. **A**
6. **A**
7. **D**
8. **B**
9. **D**
10. **C**
11. **D**

NR 2) **2.25**

NR 3) **2486**

12. **C**
13. **A**
14. **B**
15. **B**
16. **D**
17. **B**

NR 4) **90**

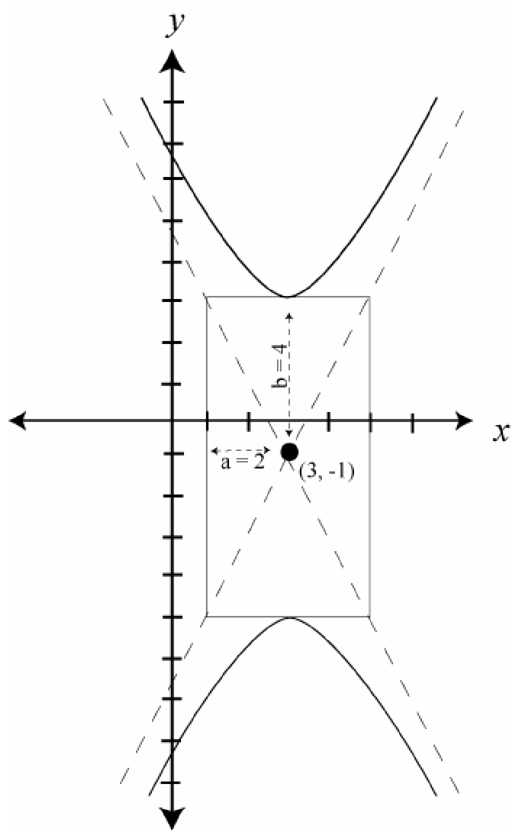
18. **D**
19. **D**
20. **D**
21. **D**
22. **A**
23. **A**
24. **A**
25. **A**
26. **C**
27. **C**
28. **A**
29. **C**

1. First identify the parameters in $\frac{(x-3)^2}{4}-\frac{(y+1)^2}{16}=-1$

The hyperbola is vertical because of the -1,

Centre is (3, -1)

a-value is 2
b-value is 4.



The range is $y \geq 3$, $y \leq -5$
The answer is **A.**

NR #1. The equation given is $x^2+m(y-70)=0$ and the point (59, 0) can be read off the graph. Plug this in for *x* and *y*, then solve for *m*.

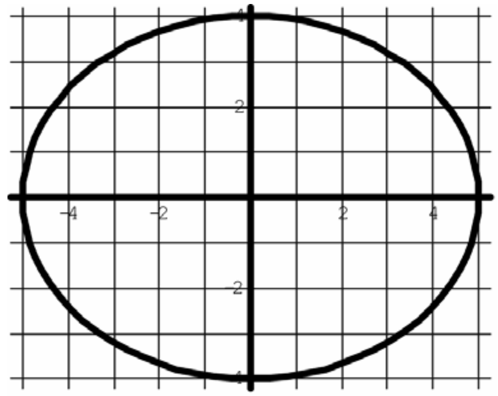
$$x^2+m(y-70)=0$$
$$59^2+m(0-70)=0$$
$$3481-70m=0$$
$$3481=70m$$
$$m=49.7$$

The answer is **49.7**

2. Two parallel lines may not be formed using a double napped cone, a cylinder is required to make this shape.
The answer is **A.**

3. The conic $16x^2 + 25y^2 - 400 = 0$ should be converted to standard form in order to properly draw the graph.

$$16x^2 + 25y^2 - 400 = 0$$
$$16x^2 + 25y^2 = 400$$
$$\frac{16x^2}{400} + \frac{25y^2}{400} = \frac{400}{400}$$
$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$



The range of the ellipse is $-4 \le y \le 4$, which is the same as saying the absolute value of range is less or equal to 4.
The answer is **B.**

4. For an ellipse, the product of *A* and *C* must be positive *(have the same sign)*.
The answer is **A.**
(B gives a hyperbola, and C & D give parabolas)

5. Convert the equation to standard form by completing the square

$$-2y^2 - x + 20y - 47 = 0$$
$$-2y^2 + 20y = x + 47$$
$$-2(y^2 - 10y \qquad) = x + 47$$
$$-2(y^2 - 10y + 25) = x + 47 - 50$$
$$-2(y - 5)^2 = x - 3$$

The vertex is located at (3, 5).
The answer is **A.**

6. The equation $3x^2 - Dx + Ey - F = 0$ has no y^2 term, so the C value is zero. This will make a parabola. The degenerate of a parabola is a single line.
The answer is **A.**

7. If the distance between x – intercepts is $2\sqrt{11}$, half of that is $\sqrt{11}$, and that is the a – value of the ellipse.
If the distance between y – intercepts is 10, half of that is 5, and that is the b – value of the ellipse.
The centre is at (0, 0)

Plugging into the standard form equation, we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{\left(\sqrt{11}\right)^2} + \frac{y^2}{(5)^2} = 1$$

$$\frac{x^2}{11} + \frac{y^2}{25} = 1$$

The answer is **D.**

8. Convert $\frac{3(x-1)^2}{17} - \frac{(y+1)^2}{17} = 1$ to general form

$$17\left(\frac{3(x-1)^2}{17} - \frac{(y+1)^2}{17}\right) = 17(1)$$

$$3(x-1)^2 - (y+1)^2 = 17$$

$$3\left(x^2 - 2x + 1\right) - \left(y^2 + 2y + 1\right) = 17$$

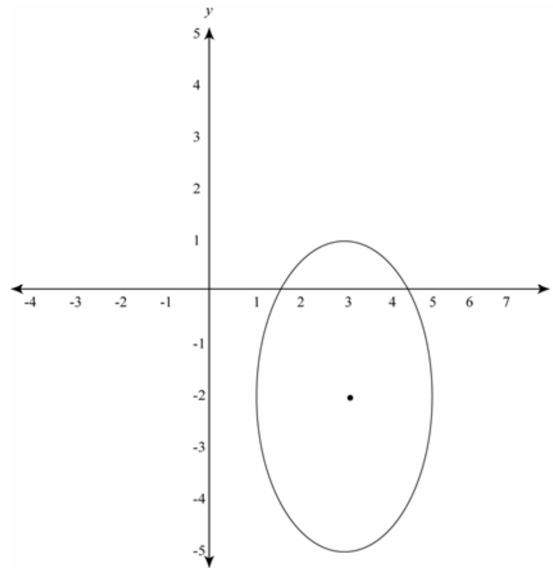
$$3x^2 - 6x + 3 - y^2 - 2y - 1 = 17$$

$$3x^2 - y^2 - 6x - 2y - 15 = 0$$

Note that $C < 0$ was extra information not needed to solve the question.

The value of E is -2.
The answer is **B.**

9. First complete the square to obtain the standard form equation.



$$9x^2+4y^2-54x+16y+61=0$$
$$9x^2-54x+4y^2+16y=-61$$
$$9\left(x^2-6x\quad\right)+4\left(y^2+4y\quad\right)=-61$$
$$9\left(x^2-6x+9\right)+4\left(y^2+4y+4\right)=-61+81+16$$
$$9(x-3)^2+4(y+2)^2=36$$
$$\frac{9(x-3)^2}{36}+\frac{4(y+2)^2}{36}=\frac{36}{36}$$
$$\frac{(x-3)^2}{4}+\frac{(y+2)^2}{9}=1$$

The range is $-5\le y\le 1$

The answer is **D.**

10. The y – intercepts of $x^2+y^2-x+5y-6=0$ can be found by substituting zero for x, then solving for y.

$$x^2+y^2-x+5y-6=0$$
$$y^2+5y-6=0$$
$$(y+6)(y-1)=0$$
$$y=-6,\ 1$$

The answer is **C.** *Remember that y-intercepts are actually a coordinate with x = 0, so write the y-intercepts as (0, -6) and (0,1)*

11. The second graph has been reflected in the line *y* = *x*, which makes it an inverse. The answer is **D.**

NR 2. If the parabola has a vertical axis of symmetry, the standard form equation is $y - k = a(x - h)^2$. Plug the data into this equation and solve for a.

h = -3, k = 5, x = -1, y = -4.

$$y - k = a(x - h)^2$$
$$-4 - 5 = a(-1 - (-3))^2$$
$$-9 = a(2)^2$$
$$-9 = 4a$$
$$a = \frac{-9}{4}$$

The answer for $|a|$ is **2.25**

NR 3. Since the total distance across the spark gap is double the value of a, we need to multiply this value by 2.

The answer is **2486** nm

12. To transform the x-intercept of 4 to 6, multiply by 1.5, or $\frac{3}{2}$

To transform the y-intercept of 4 to 10, multiply by 2.5, or $\frac{5}{2}$

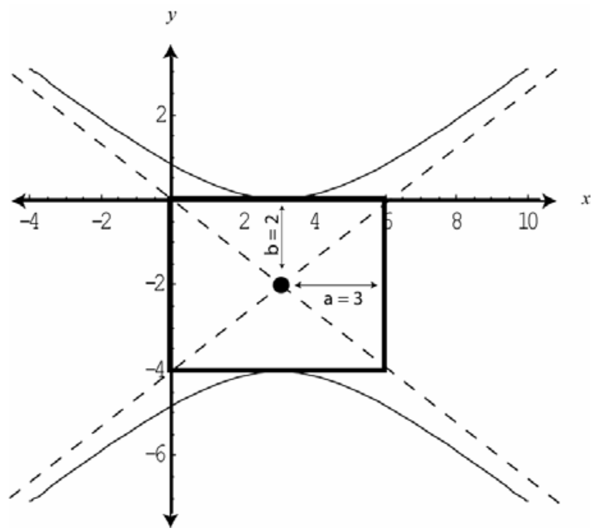
The answer is **C.**

13. First draw in the reference box to determine the *a* and *b* values

$a = 3$
$b = 2$
Centre = (3, -2)

Plug these into the standard form equation for a vertical hyperbola

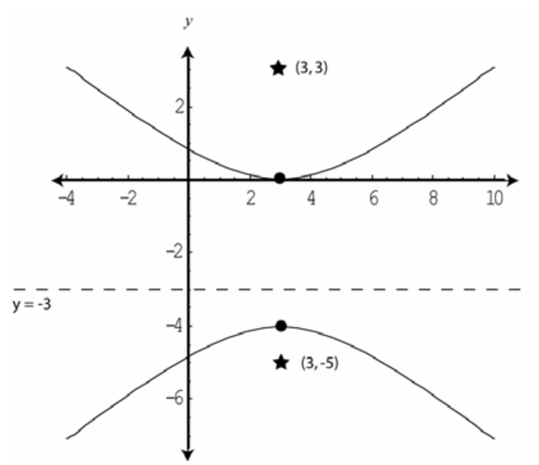
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = -1$$
$$\frac{(x-3)^2}{3^2} - \frac{(y-(-2))^2}{2^2} = -1$$
$$\frac{(x-3)^2}{9} - \frac{(y+2)^2}{4} = -1$$



The answer is **A.**

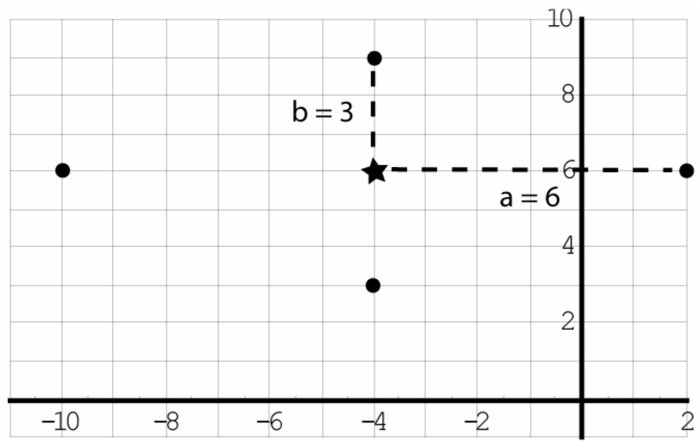
14. Draw in the line $y = -3$, and then perform the stretch by a factor of 2 on the vertices. The new vertices can be found at (3, 3) and (3, -5)

The new range is $y \le -5$, $y \ge 3$
The answer is **B.**



15. If both A and C are negative, the product will give a positive, making the shape an ellipse.
The answer is **B.**

16. Draw in the known points, and use them to determine the a and b values.



The standard form of the ellipse is $\frac{(x+4)^2}{36}+\frac{(y-6)^2}{9}=1$

The answer is **D.**

17. Since the plane is cutting between the generator and central axis, it will give a hyperbola.
The answer is **B.**

NR 4. The cutting plane can produce a circle at only one angle, and that is when it is perpendicular to the central axis.
The answer is **90˚**

18. The plane passing parallel to the generator will give a parabola, and since it passes through the vertex, it will give the degenerate case of a single line.
The answer is **D.**

19. Stretches must be applied before translations, so this rules out both A and B, and C will not give the correct shape.
The answer is **D.**

20. Replace *x* with (*x* + 2) to represent 2 units left, and replace *y* with (*y* + 1) to represent 1 unit down.

$$2(x+2)^2+(y+1)^2-2(x+2)+3(y+1)-9=0$$
$$2\left(x^2+4x+4\right)+\left(y^2+2y+1\right)-2(x+2)+3(y+1)-9=0$$
$$2x^2+8x+8+y^2+2y+1-2x-4+3y+3-9=0$$
$$2x^2+y^2+6x+5y-1=0$$

The answer is **D.**

21. First state what you know, then plug the values into the standard form equation

$$a=\sqrt{10}b, \quad b=b, \quad h=2, \quad k=-1$$

$$\frac{(x-2)^2}{\left(\sqrt{10}b\right)^2}+\frac{(y+1)^2}{b^2}=1$$
$$\frac{(x-2)^2}{10b^2}+\frac{(y+1)^2}{b^2}=1$$

The answer is **D**.

22. If the vertex angle is 60˚, half of that will give the generator angle, which is 30˚. An ellipse will be formed if the angle of the cutting plane is between the generator and perpendicular, so $30^0 < x < 90^0$. Note that you don't want to use the equal sign, since that would give a parabola & circle too.
The answer is **A.**

23. Complete the square.

$$x^2+9y-2x-35=0$$
$$x^2-2x=-9y+35$$
$$x^2-2x+1=-9y+35+1$$
$$(x-1)^2=-9y+36$$
$$(x-1)^2=-9(y-4)$$

The vertex is located at (1, 4)
The answer is **A.**

24. Replace x with $x + h$ and y with $y - k$ to account for the transformations.

$x^2 + y^2 = 1$

$(x+h)^2 + (y-k)^2 = 1$

To find the x-intercepts, let $y = 0$.

$(x+h)^2 + (0-k)^2 = 1$

$(x+h)^2 + k^2 = 1$

The answer is **A.**

25. The standard form of $x - h = a(y-k)^2$ will give a horizontal parabola since the y is squared. A horizontal parabola does not pass the vertical line test, so it is not a function. The answer is **A.**

26. Complete the square to write the conic in standard form.

$x^2 + y^2 + 6x - 2y - 6 = 0$

$x^2 + 6x + y^2 - 2y = 6$

$\left(x^2 + 6x + 9\right) + \left(y^2 - 2y + 1\right) = 6 + 9 + 1$

$\left(x+3\right)^2 + \left(y-1\right)^2 = 16$

The radius is $\sqrt{16} = 4$ units.
The answer is **C.**

27. First draw in the reference box for the hyperbola to determine the *a* and *b* values.

From here, we can see that

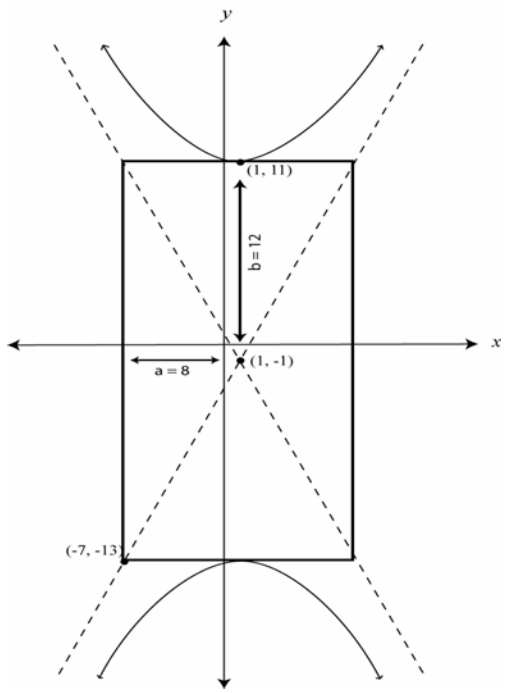
$a = 8$
$b = 12$
Centre = (1, -1)

Plug these values into the standard form equation of a vertical hyperbola.

$$\frac{(x-1)^2}{8^2} - \frac{(y+1)^2}{12^2} = -1$$

$$\frac{(x-1)^2}{64} - \frac{(y+1)^2}{144} = -1$$

The answer is **C.**



28. From the standard form $\frac{(x-2)^2}{16} + \frac{(y+3)^2}{36} = 1$, the centre is at (2, -3)

If moved 3 left and 5 up, the new centre is (-1, 2)

The new standard form, then, is $\frac{(x+1)^2}{16} + \frac{(y-2)^2}{36} = 1$

The answer is **A.**

29. Multiply both sides of the equation by the common denominator.

$$400\left[\frac{(x-1)^2}{16} + \frac{(y-1)^2}{25}\right] = 400[1]$$

$$25(x-1)^2 + 16(y-1)^2 = 400$$

$$25\left(x^2 - 2x + 1\right) + 16\left(y^2 - 2y + 1\right) = 400$$

$$25x^2 - 50x + 25 + 16y^2 - 32y + 16 = 400$$

$$25x^2 + 16y^2 - 50x - 32y - 359 = 0$$

The answer is **C.**

Written Response 1

Ellipse:
First state what you know: a = 60, b = 40, centre = (0, 0)
Plug these values into the standard form equation to get the answer.

The equation is $\frac{x^2}{3600}+\frac{y^2}{1600}=1$

Parabola:
First state what you know: Vertex = (0, 40),
and a point on the graph is (60, 0)
Plug these values into the standard form equation to solve for a, then you can write the final equation.

$$y-k=a(x-h)^2$$
$$0-40=a(60-0)^2$$
$$-40=a\left(60\right)^2$$
$$-40=3600a$$
$$a=-\frac{1}{90}$$

The equation is $y-40=-\frac{1}{90}x^2$

The height at x = 30 gives the following for each model:

Ellipse: 34.64 m
Parabola: 30 m

The parabola comes closest to the actual height.

Written Response 2

Dartboard: The diameter of the dartboard is 50.3 cm, so the radius is 25.15 cm

$x^2 + y^2 = (25.15)^2$

$x^2 + y^2 = 632.52$

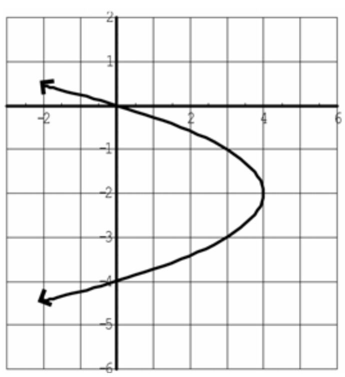
Backing: The diameter of the dartboard is 80.7 cm, so the radius is 40.35 cm

$x^2 + y^2 = (40.35)^2$

$x^2 + y^2 = 1628.12$

Transformation: The small circle undergoes a horizontal stretch by a factor of $\frac{40.35}{25.15} = 1.6$, and a vertical stretch by the same value.

Written Response 3



The vertex is located at (4, -2). These are the *h* & *k* values.

A point on the graph is located at (0, 0) This is a value you can use for *x* & *y*.

Plug everything in and solve for *a*.

Standard form for horizontal parabolas $\rightarrow x - h = a(y - k)^2$

$0 - 4 = a(0 - (-2))^2$

$0 - 4 = a(0 + 2)^2$

$-4 = a(4)$

$a = -1$

The equation is: $x - 4 = -(y + 2)^2$

x – intercept:

Set y = 0, then solve for x.

$x - 4 = -(0 + 2)^2$

$x - 4 = -4$

$x = 0$

y – intercepts:

Set x = 0, then solve for y.

$0 - 4 = -(y + 2)^2$

$-4 = -(y + 2)^2$

$4 = (y + 2)^2$

$\sqrt{4} = \sqrt{(y + 2)^2}$

First evaluate for $+2$

$+2 = y + 2$

$y = 0$

Then evaluate for -2

$-2 = y + 2$

$y = -4$

The domain is $x \le 4$

The range is $y \in R$